Perspectives in Topological Phases

Quy Nhon, Vietnam
July 17, 2018

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Part I

A clean phase transition from Composite Fermi sea to Moore-Read in bilayer graphene

Part II

Shear sound of 2D Fermi liquids

Phase transition of Composite Fermions in BLG

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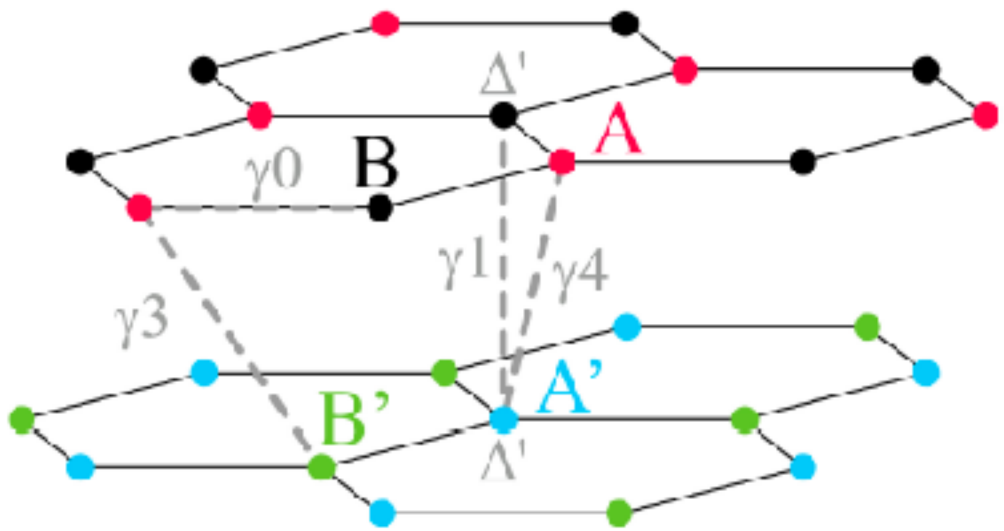
- Clean phase transition between Pfaffian and composite fermi liquid in bilayer graphene by tuning perpendicular magnetic field.

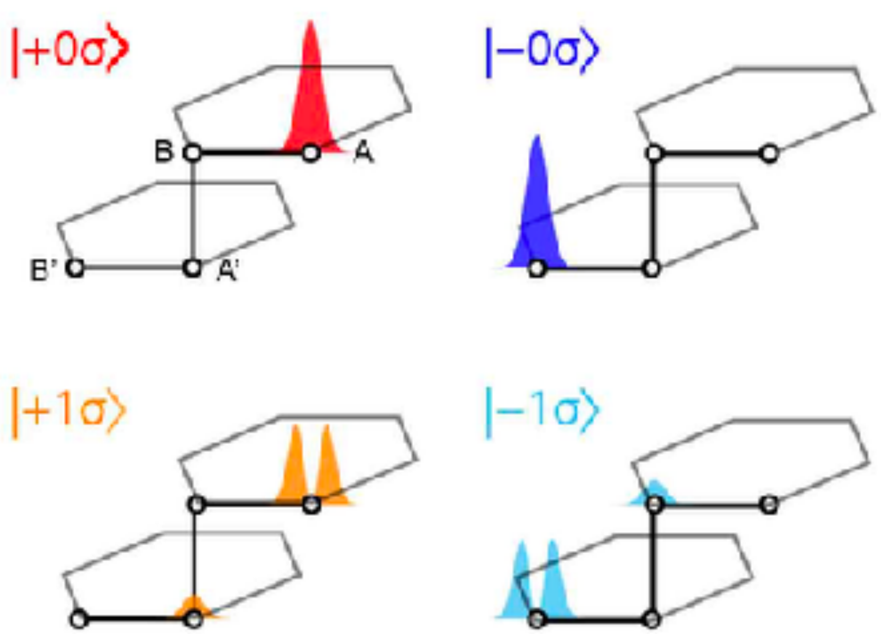
Quantum Hall in Bilayer Graphene



$$
\begin{aligned}
\gamma_0 &= -2.61\text{eV} \\
\gamma_1 &= .361\text{eV} \\
\gamma_4 &= .138\text{eV} \\
\Delta' &= .015\text{eV}
\end{aligned}
$$

Jung, & MacDonald, PRB (2014)

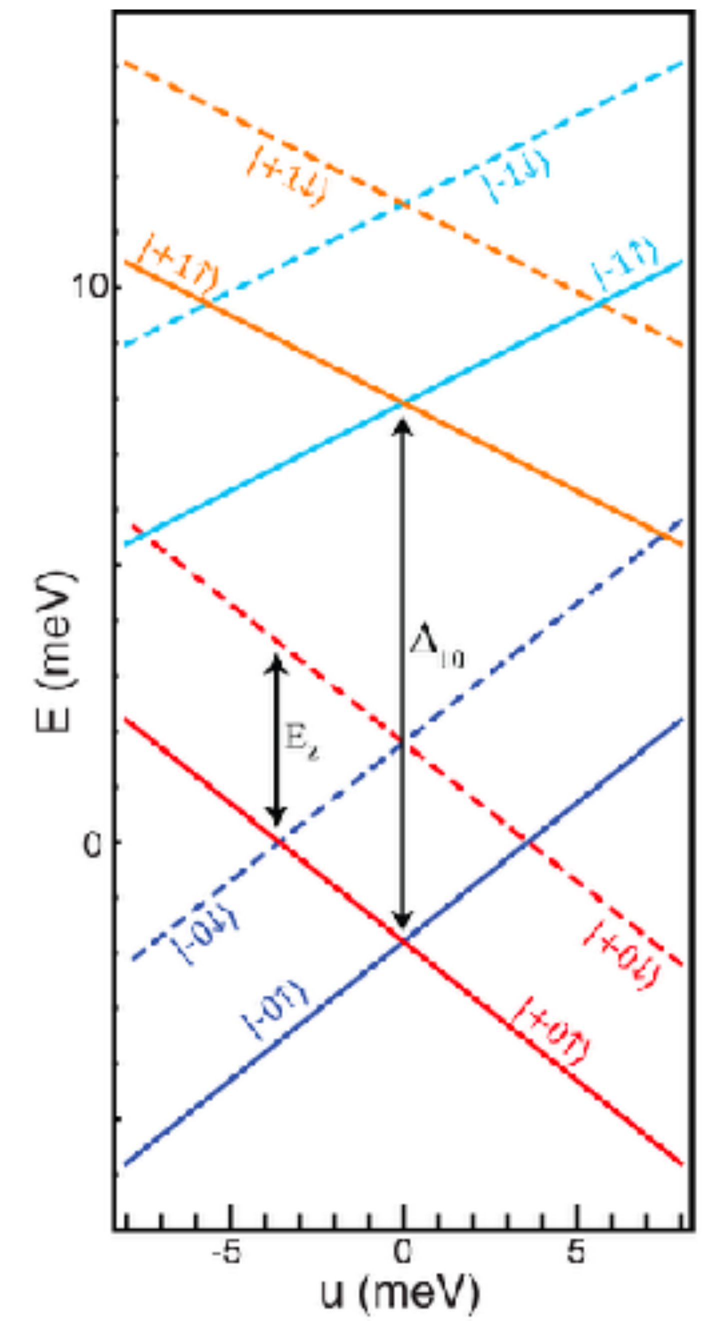
Nearly 8-fold degenerate LL



$\Delta_{10} \sim 10meV$

at *B = 10T*

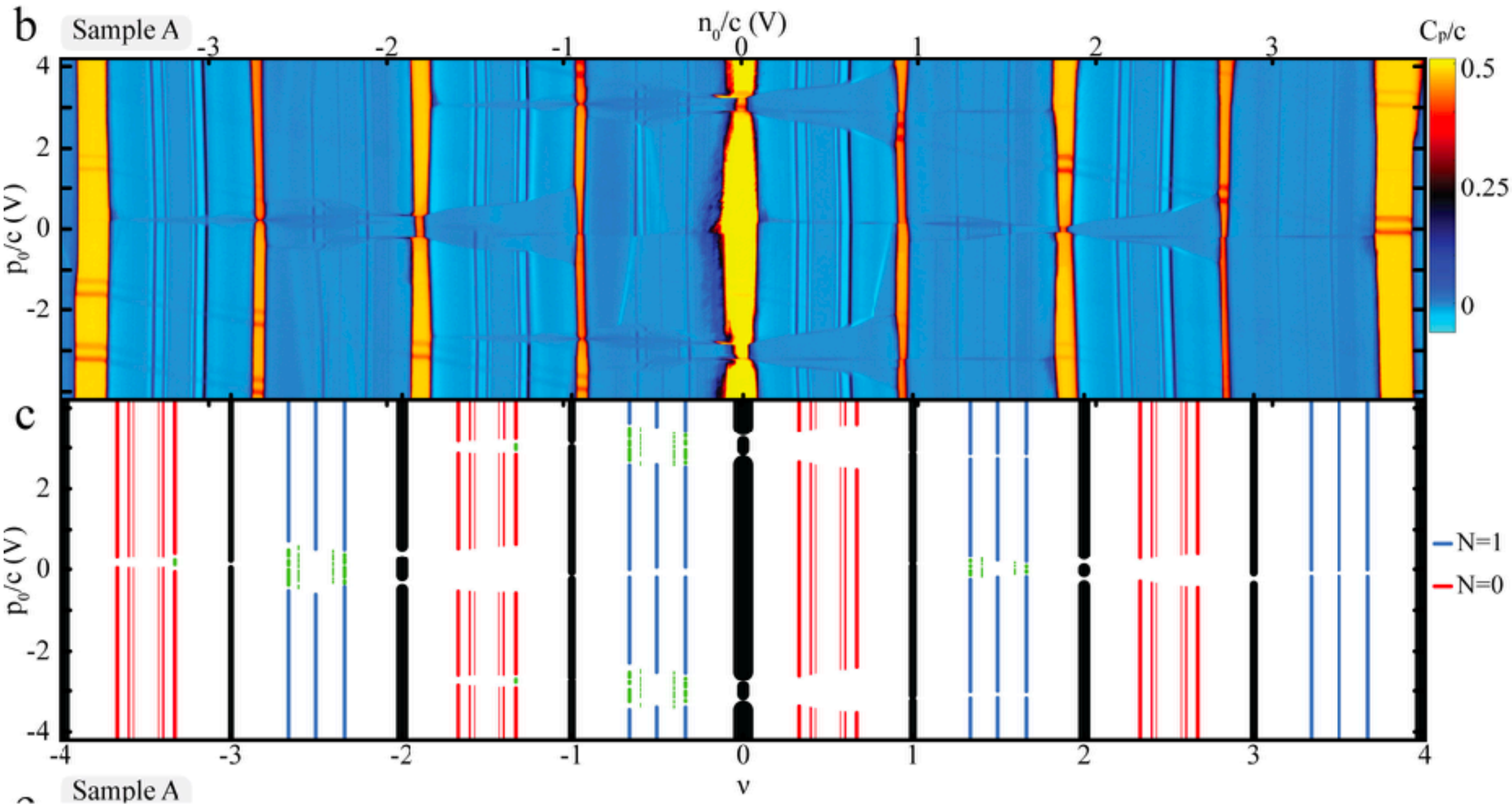
Single particle splitting of LLs



Hunt et al. Nat. Comms. (2017)

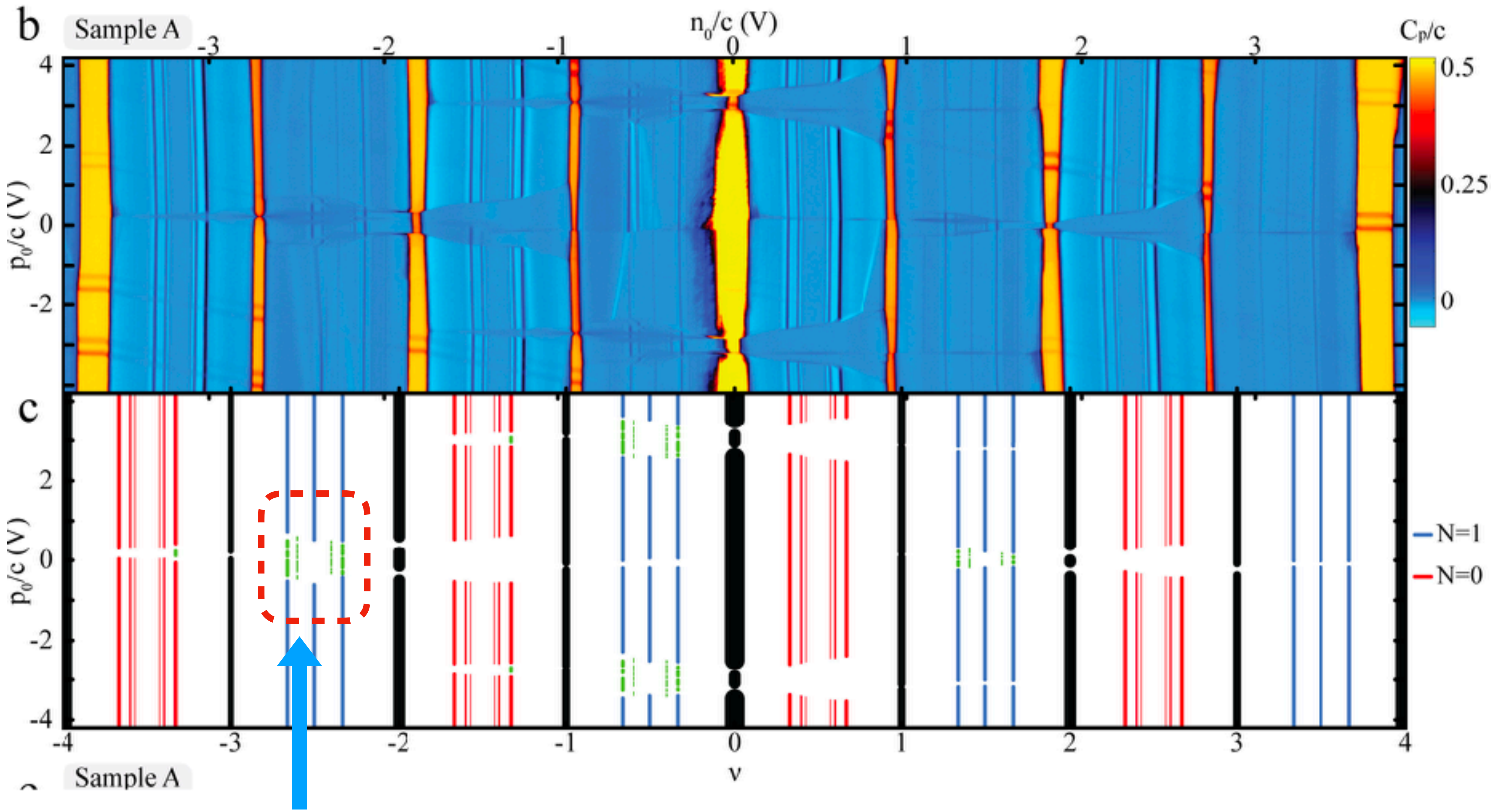
Quantum Hall in Bilayer Graphene

Zibrov et al. Nature 2017



Quantum Hall in Bilayer Graphene

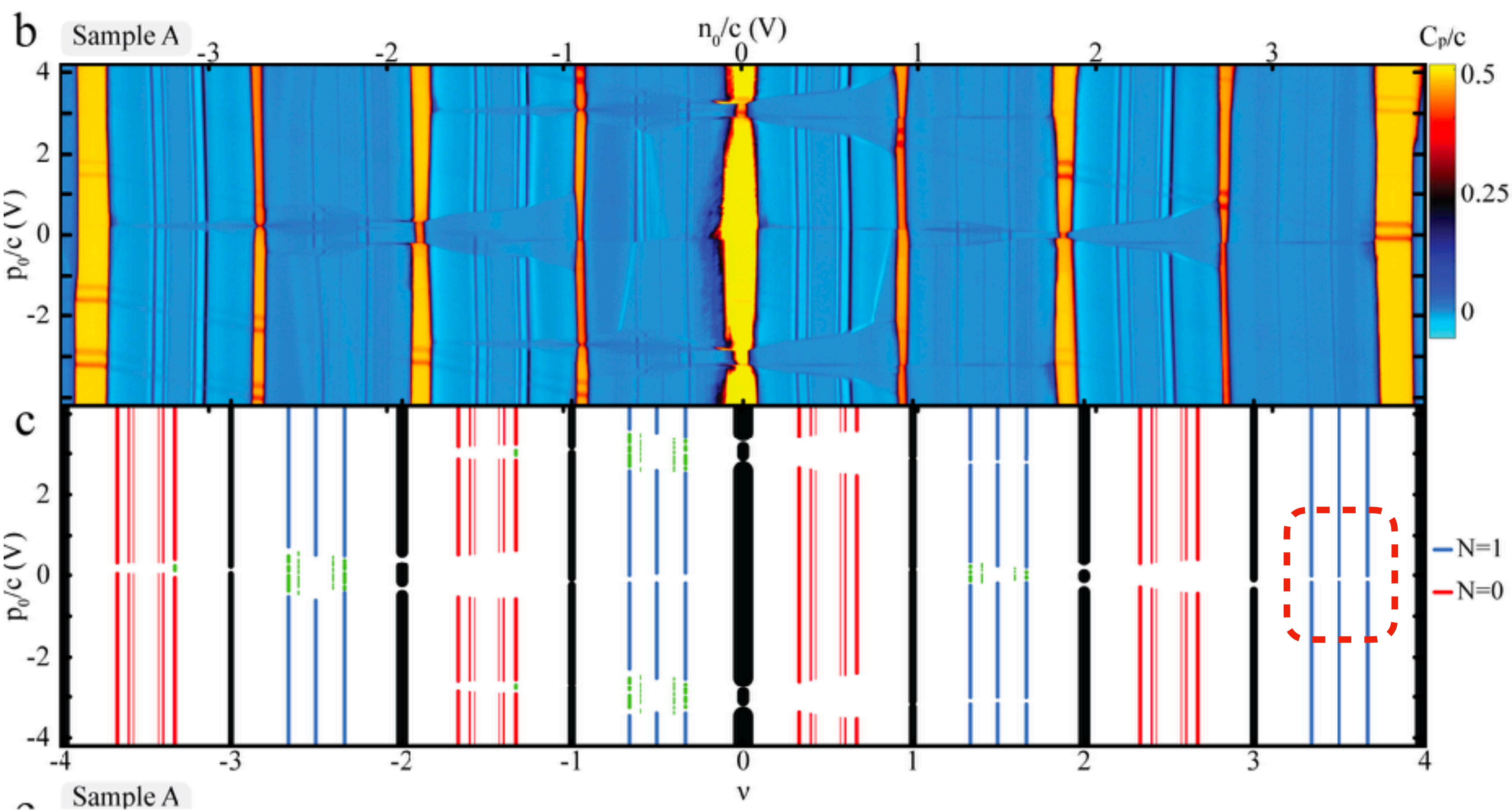
Zibrov et al. Nature 2017



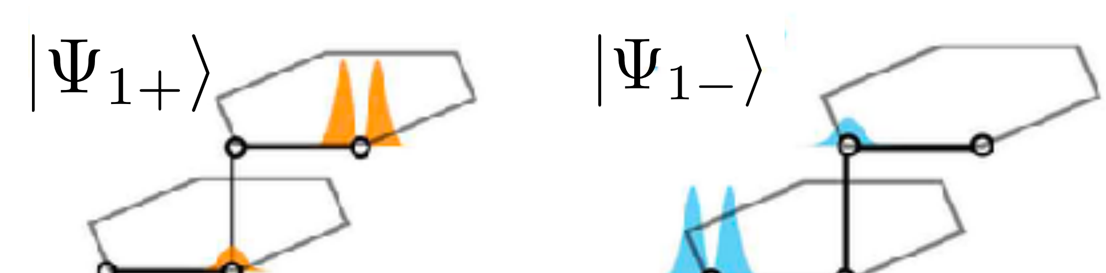
Zaletel et al. arXiv:1803.08077
Barkeshli et al. PRL 2018

Quantum Hall in Bilayer Graphene

Zibrov et al. Nature 2017

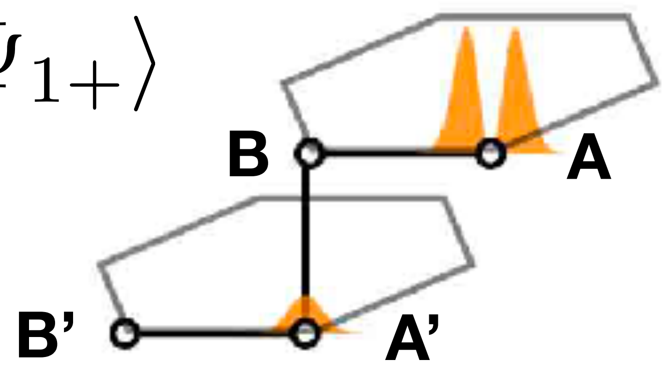


Near SU(2) symmetry for two N=1 orbitals



Quantum Hall in Bilayer Graphene

$|\Psi_{1+}\rangle$

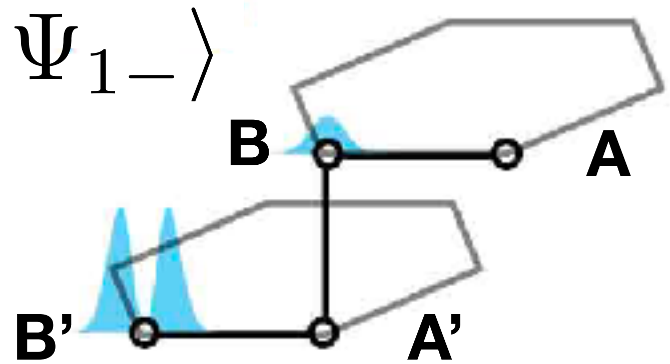


N=1 orbitals have mixed cyclotron character

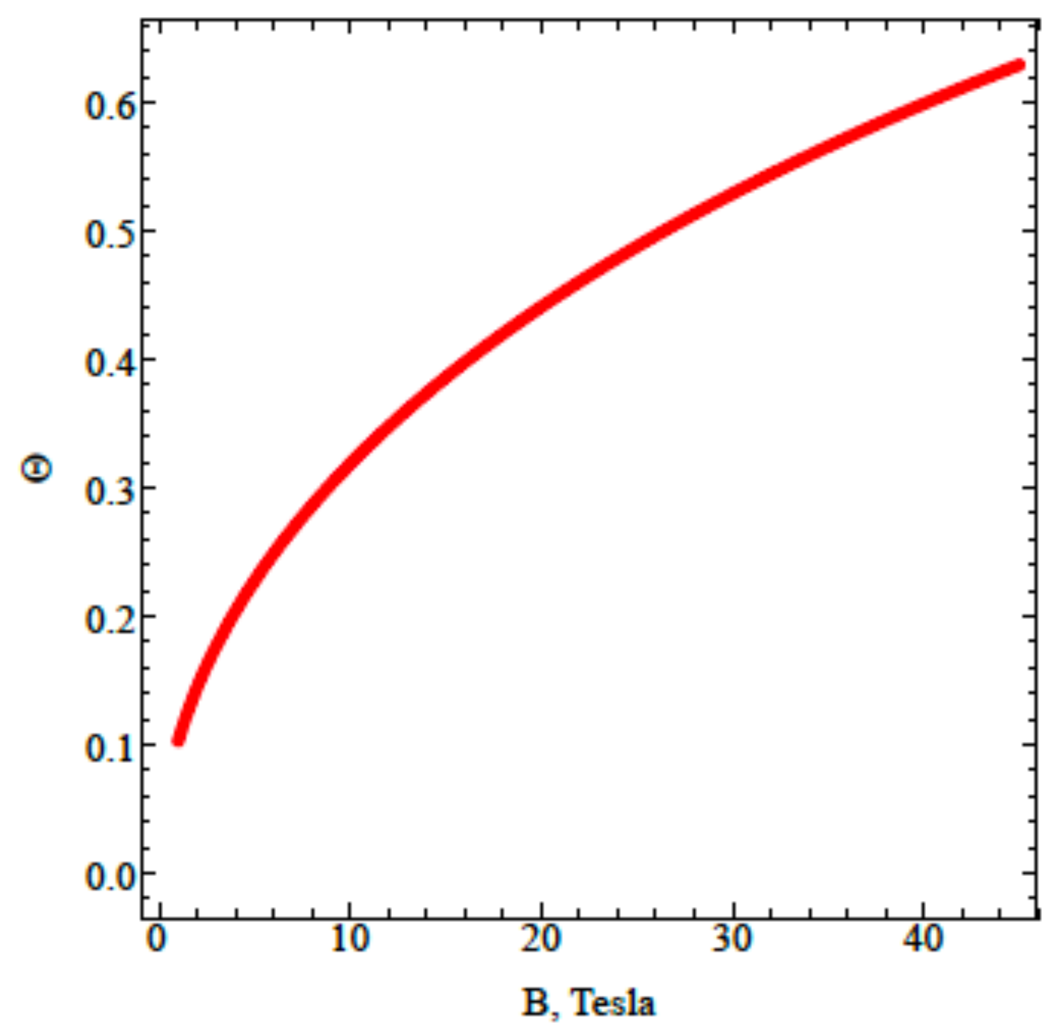
$$|\Psi_{1+}\rangle = \sin(\Theta)|n=0, A'\rangle + \cos(\Theta)|n=1, A\rangle$$

$$|\Psi_{1-}\rangle = \sin(\Theta)|n=0, B\rangle + \cos(\Theta)|n=1, B'\rangle$$

$|\Psi_{1-}\rangle$



Zibrov et al. Nature 2017



At B=40T

$\sin(\Theta)^2 \sim 0.32$

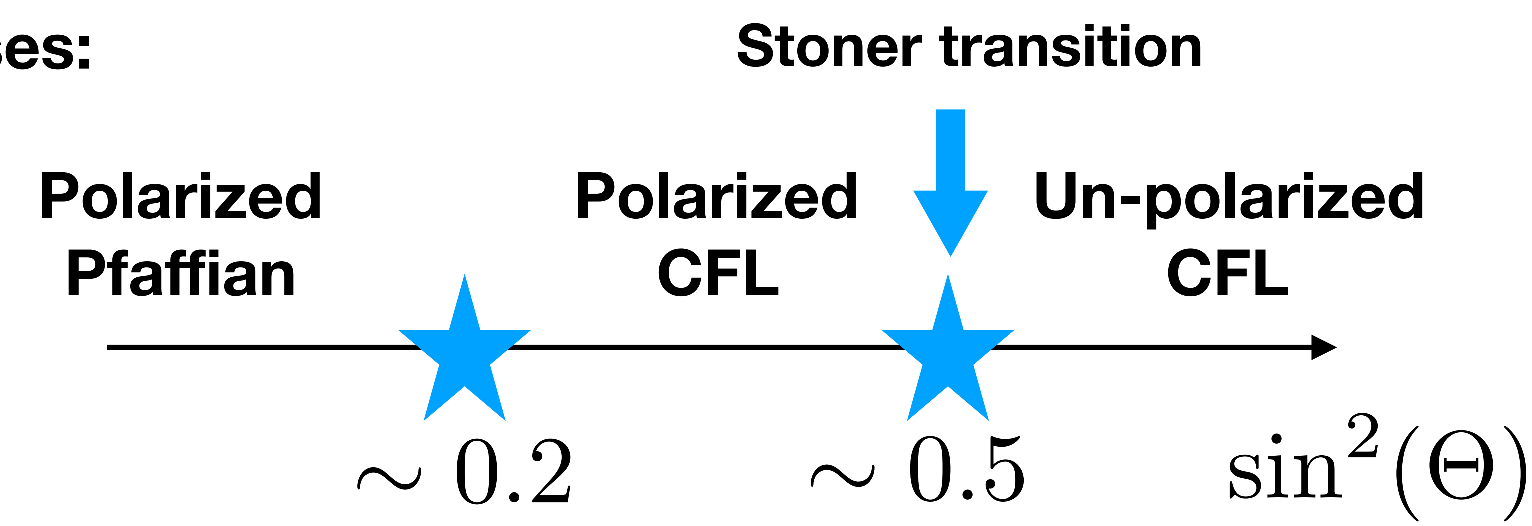
Ideal platform to realize Moore-Read to CFL transition

$$|\Psi_{1\sigma}\rangle = \sin(\Theta)|n=0,B,\sigma\rangle + \cos(\Theta)|n=1,B',\sigma\rangle$$

Ideal Hamiltonian has SU(2) symmetry between $\sigma = \{+,-\}$

$$V = P_0 \sum_{ij} \frac{e^2}{|r_i - r_j|} P_0$$

We find three phases:



Rezayi and Haldane et al. PRL 2000

Papic et al. PRB 2011

Apalkov et al. PRL 2011

Metlitskii et al. PRB 2015

Wave function of Composite Fermi Liquid

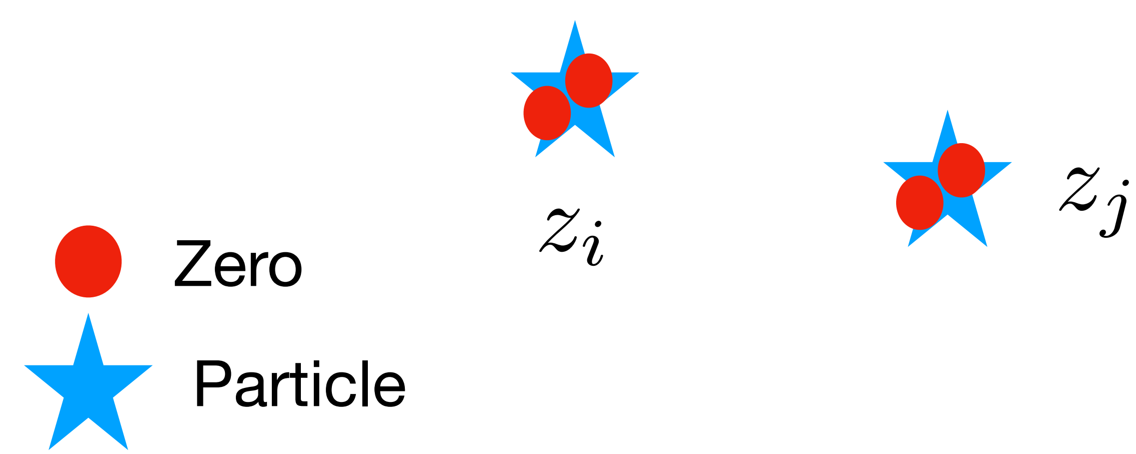
The key to quantum Hall energetics

How to make particles stay as far way from each other as possible within a Landau level at filling ν:

$$\Psi = \prod_{i<j} (z_i - z_j)^{\frac{1}{\nu}} \qquad z = x + iy$$

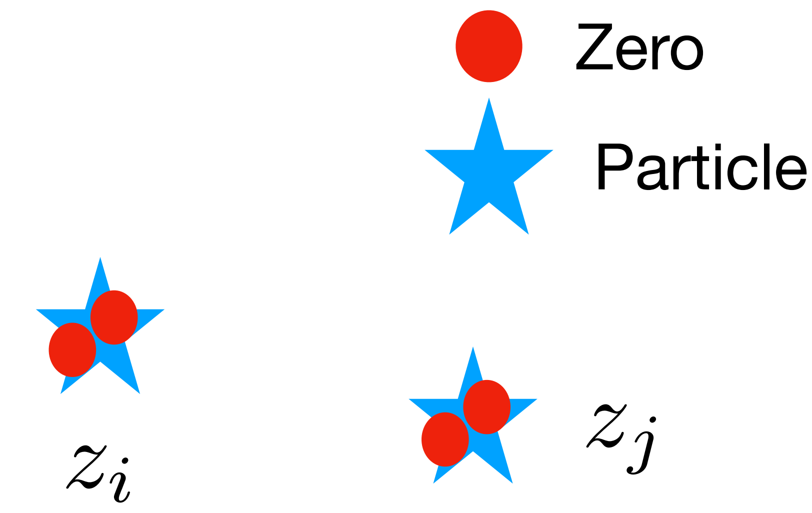
We get the bosonic Laughlin state at $\nu = \frac{1}{2}$

$$\Phi_{bose} = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$



Wave function of Composite Fermi Liquid

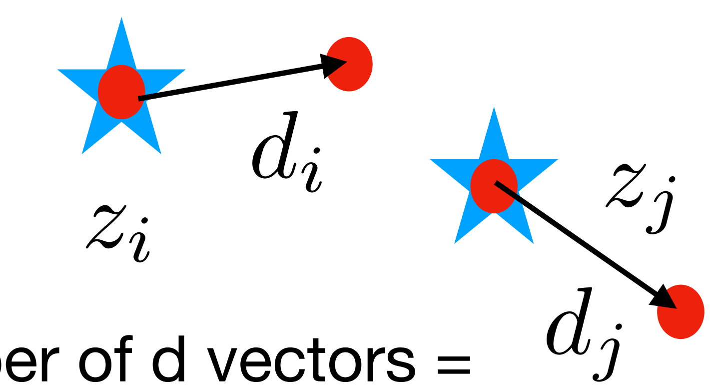
We get the bosonic Laughlin state at $\nu = \frac{1}{2}$

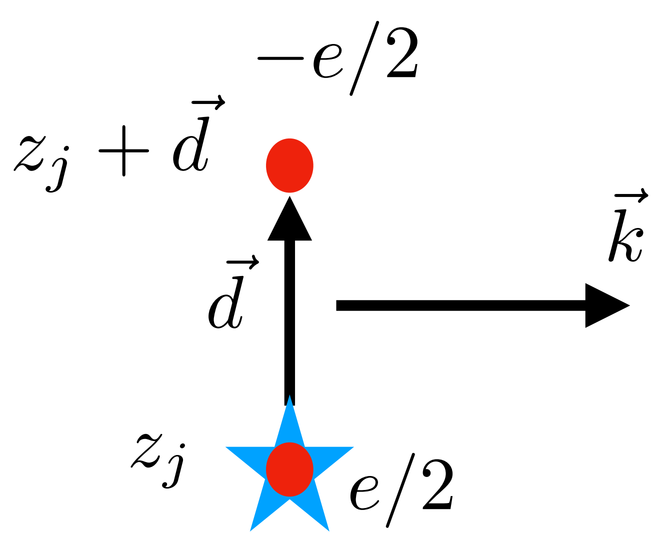


$$\Phi_{bose} = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$

To get fermions: displace the zeroes as little as possible to be able to anti-symmetrize

$$\Psi_{fermi} = \mathcal{A} \left(\prod_{i<j} (z_i - z_j)(z_i - z_j - d_i - d_j) e^{-\frac{|z_i|^2}{4l^2}} \right)$$





$$\mathbf{d}_i = -l^2 \hat{z} \times \mathbf{k}_i$$

Number of d vectors = Number of particles

N. Read, Semiconductor Science and Technology, 9, 1859 (1994).

J. K. Jain, Phys. Rev. Lett. 63, 199 (1989)

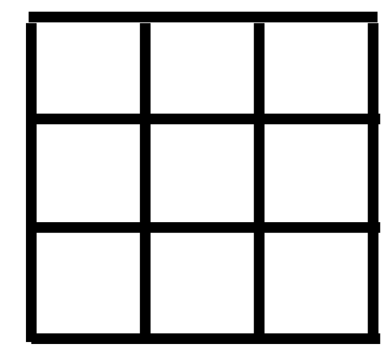
E. Rezayi and N. Read, Phys. Rev. Lett. 72, 900 (1994).

C. Wang and T. Senthil, Phys. Rev. B 94, 245107 (2016)

Composite fermions on Torus

Single particle translations form a discrete lattice on a finite size torus

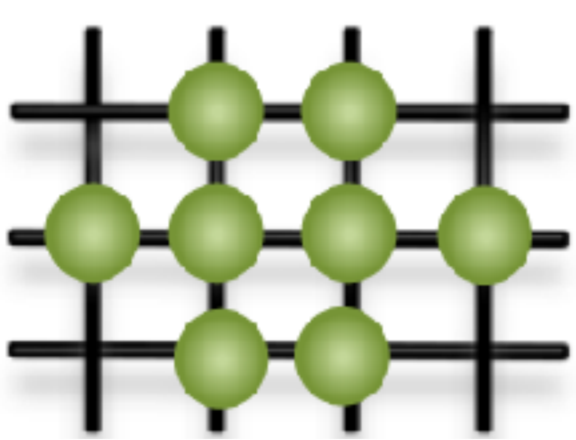
$$\mathbf{d} \in \frac{m_1\mathbf{L_1} + m_2\mathbf{L_2}}{N_\phi},\ m_{1,2} \in \mathbb{Z}\mathrm{mod}(N_\phi)$$



$\hat{t}_i(\mathbf{d})$ Translation of particle i by vector **d**

For any given set of N distinct $\{\mathbf{d}_i\}$

We can construct a Composite Fermion trial wave-function



$$|\Psi_{\text{CFL}}(\{\mathbf{d}_i\})\rangle = \det(\hat{t}_j(\mathbf{d}_i))|\Phi^{\text{Bose}}_{1/2}\rangle$$

$$\mathbf{d}_i = -l^2\hat{z} \times \mathbf{k}_i$$

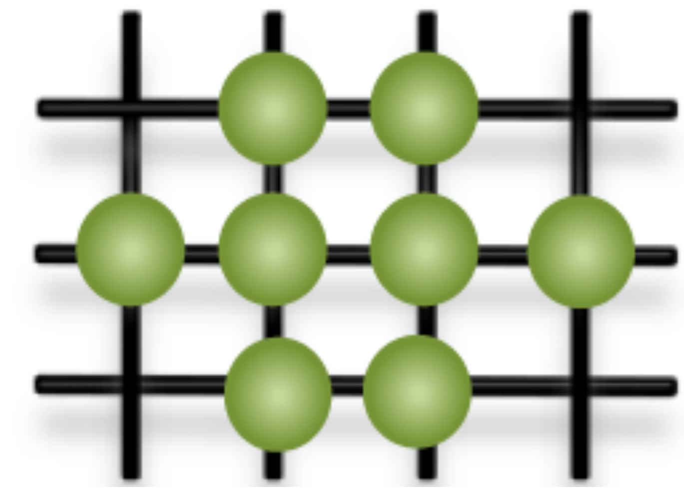
E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000).

Shao, Kim, Haldane, & Rezayi, PRL (2015).

Composite fermions on Torus

$$|\Psi_{\mathrm{CFL}}(\{\mathbf{d}_i\})\rangle = \det(\hat{t}_j(\mathbf{d}_i))|\Phi_{1/2}^{\mathrm{Bose}}\rangle, \ \mathbf{d}_i \equiv -l^2\hat{\mathbf{z}} \times \mathbf{k}_i$$

$$\mathbf{d} \in \frac{m_1\mathbf{L_1} + m_2\mathbf{L_2}}{N_\phi}, \ m_{1,2} \in \mathbb{Z}\mathrm{mod}(N_\phi)$$



Given a set of occupied states we can predict many body momenta

$$\mathbf{K} = \frac{\hat{\mathbf{z}}}{l^2} \times \sum_i \mathbf{d}_i = 2\pi\left(-\frac{\sum_i m_{2i}}{L_1}, \frac{\sum_i m_{1i}}{L_2}\right)$$

Composite fermion kinetic energy

$$E[\{\mathbf{k}_i\}] = E_0 + \frac{1}{N_e}\sum_{i<j}\epsilon(\mathbf{k}_i - \mathbf{k}_j)$$

$$\approx E_0 + \frac{1}{N_e}\sum_{i<j}\frac{|\mathbf{k}_i - \mathbf{k}_j|^2}{2m^*},$$

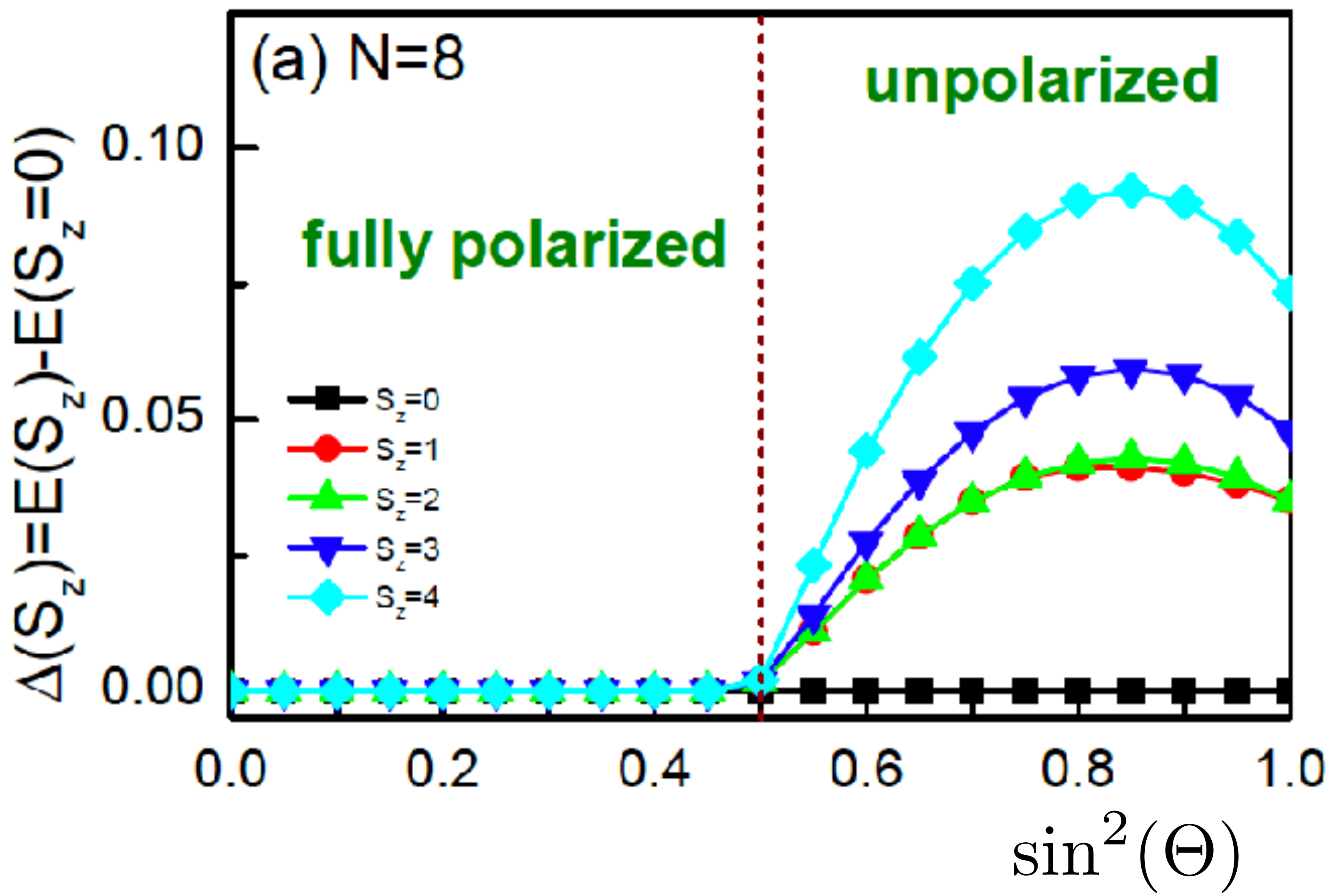
Shao, Kim, Haldane, & Rezayi, PRL (2015).

SU(2) magnet to SU(2) singlet

2 components continuously rotating from N=0 LL into a N=1LL

$$|\Theta, \sigma\rangle = \sin(\Theta)|0, B, \sigma\rangle + \cos(\Theta)|1, B'\sigma\rangle$$

ED Spectrum
Torus

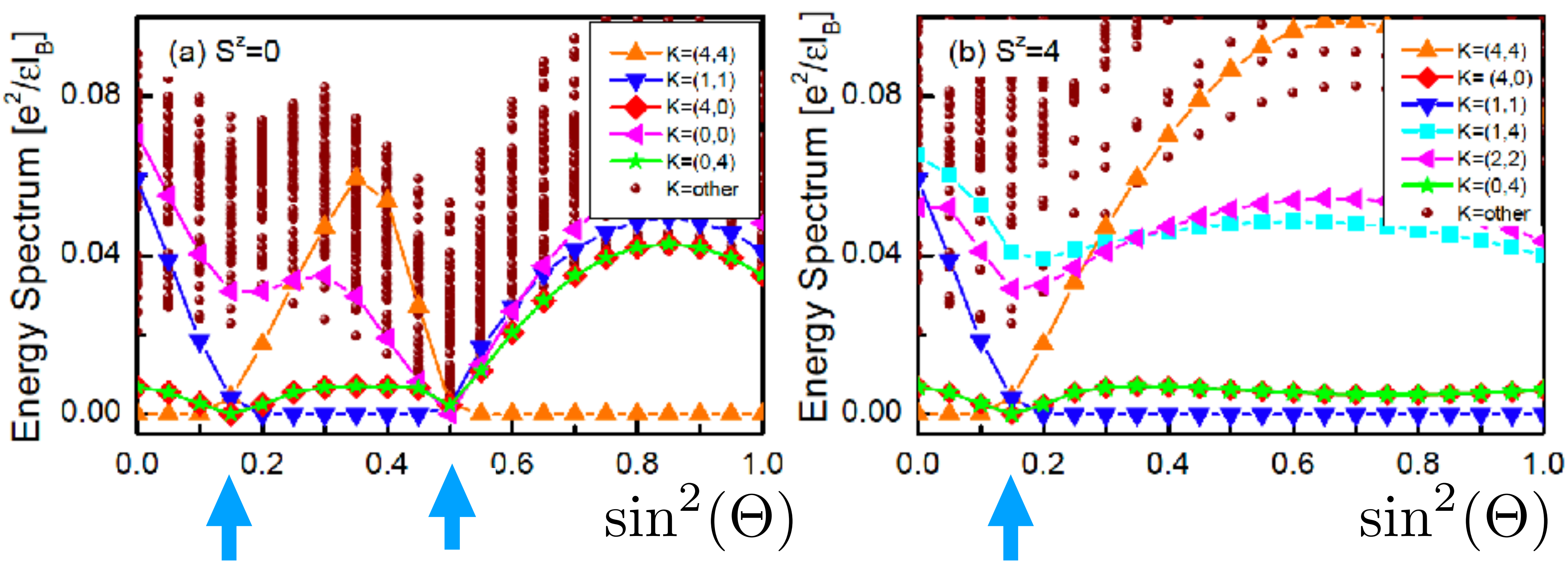


Phase transition of polarized states

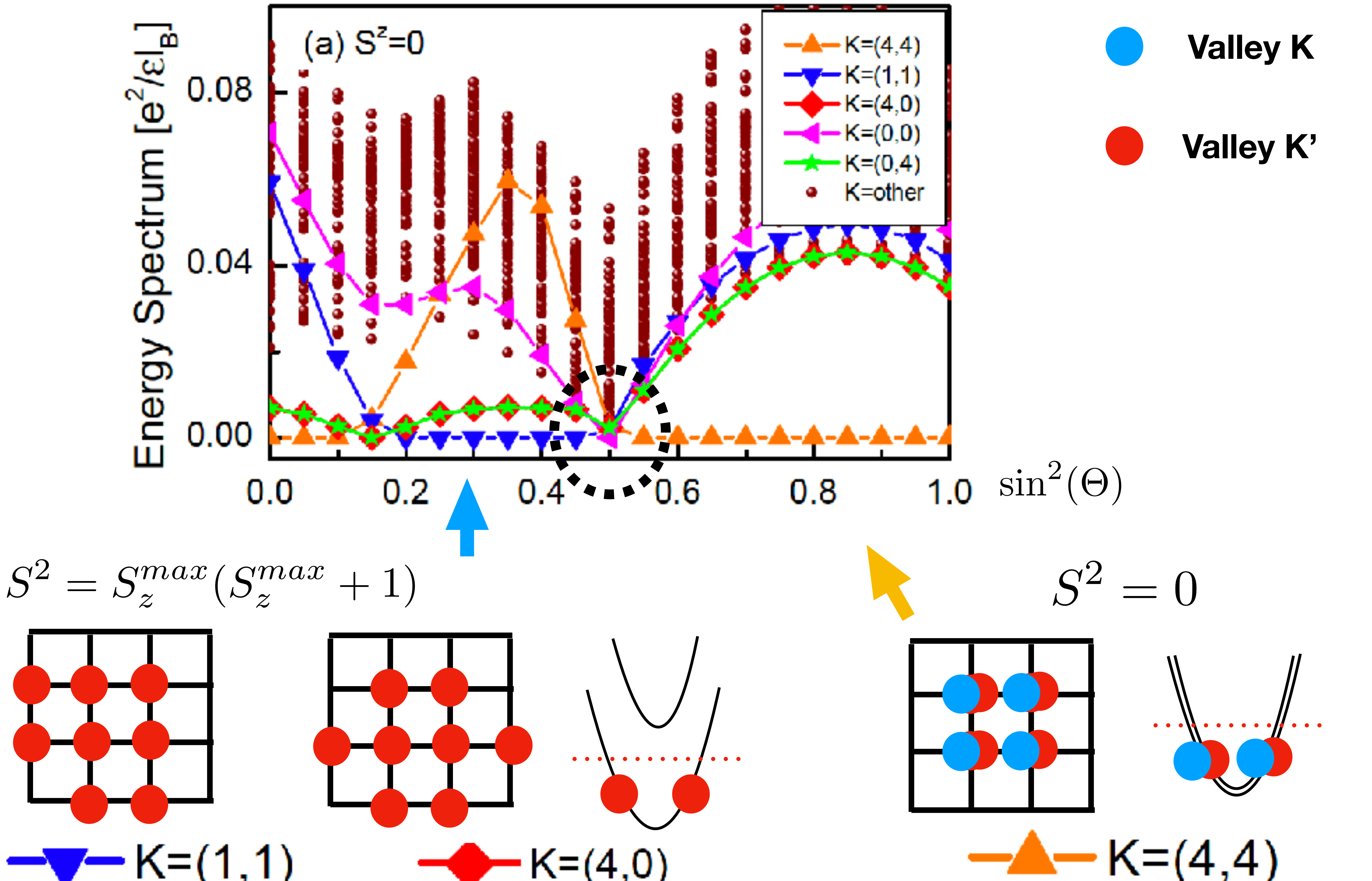
2 components continuously rotating from N=0 LL into a N=1LL

$$|\Theta, \sigma\rangle = \sin(\Theta)|0, B, \sigma\rangle + \cos(\Theta)|1, B'\sigma\rangle$$

ED Spectrum Torus



Stoner transition of CFL states



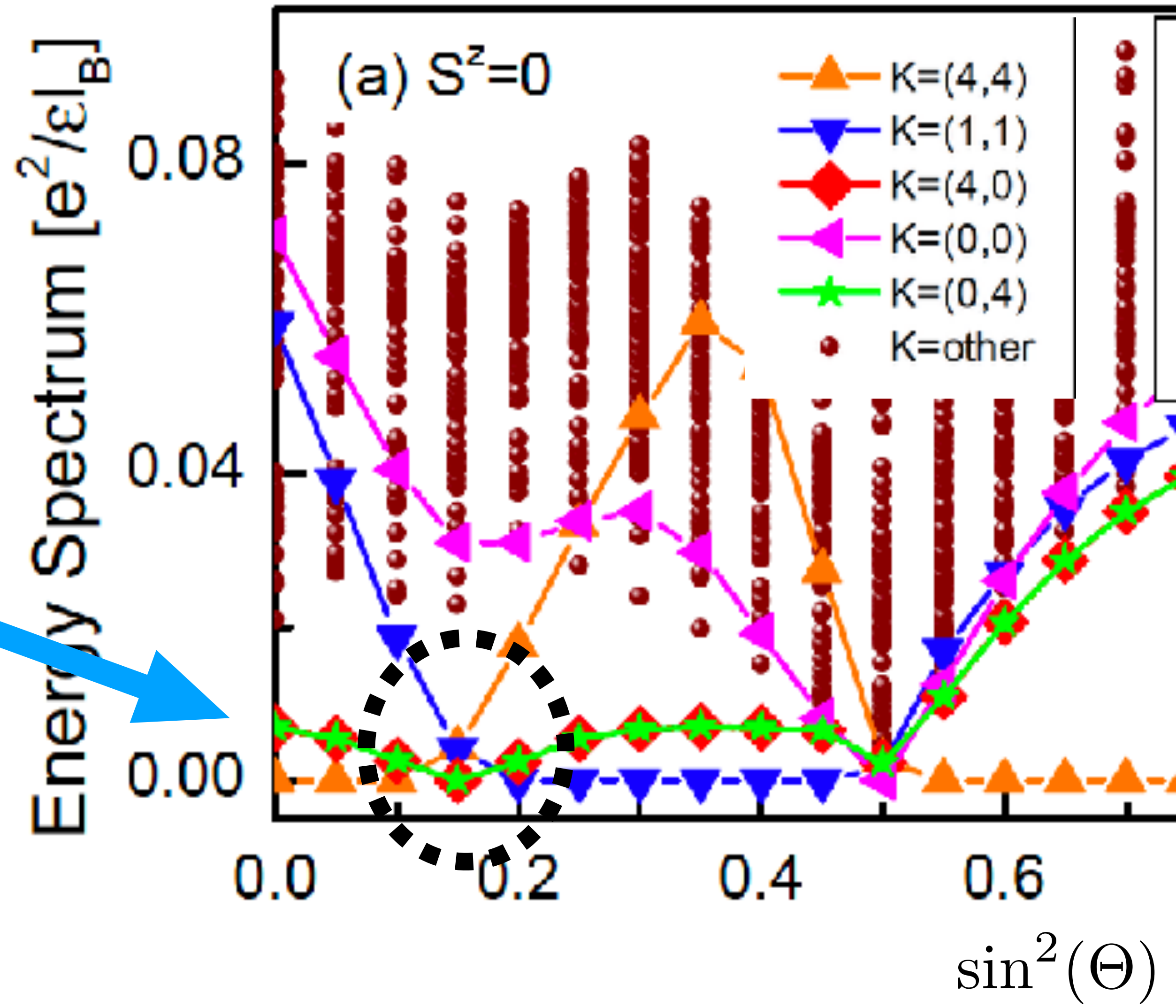
Pfaffian to CFL transition

3-fold (x 2) topological degeneracy of Moore-Read state

K=(4,4) (π, π)

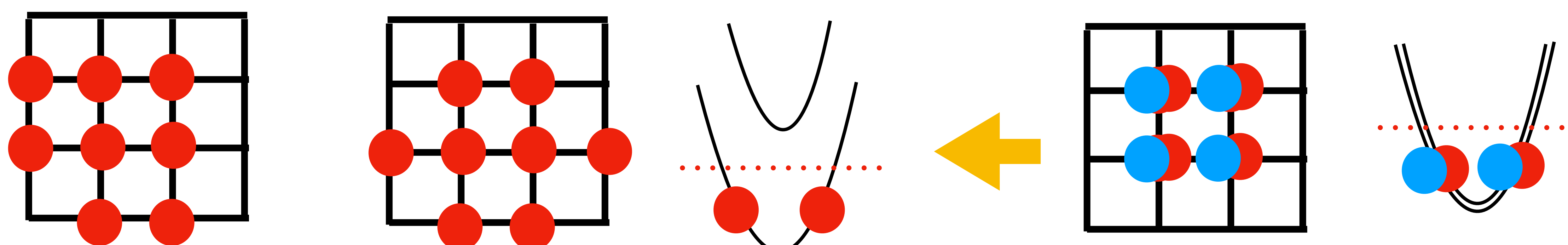
K=(4,0) $(\pi, 0)$

K=(0,4) $(0, \pi)$



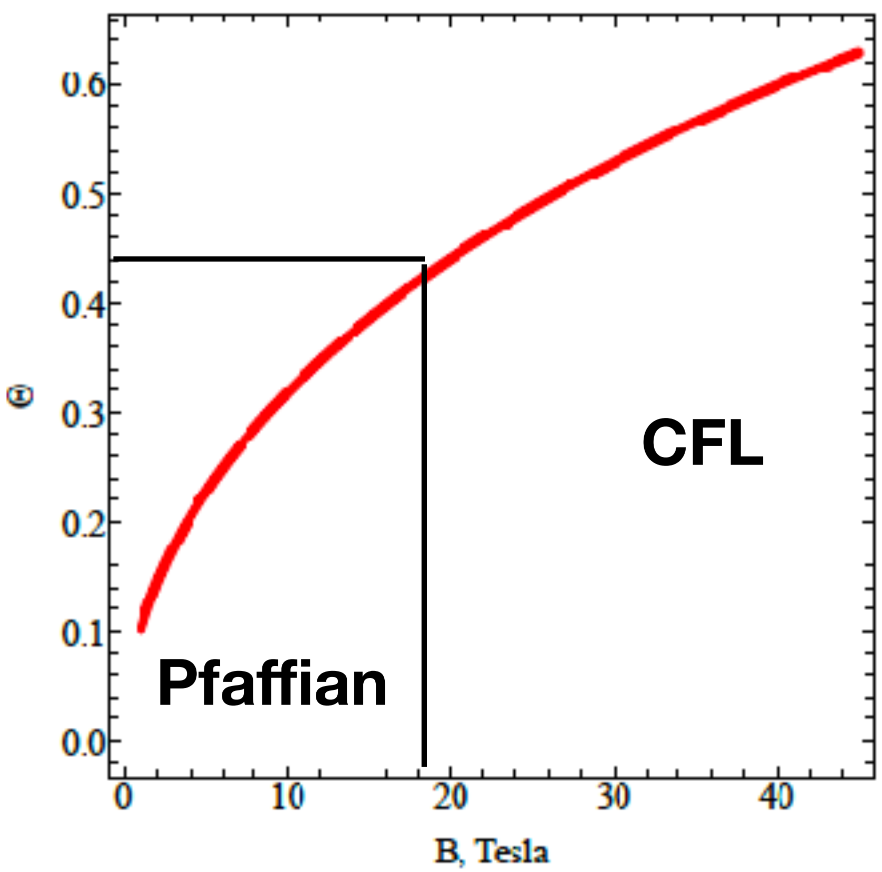
$S^2 = S_z^{max}(S_z^{max} + 1)$

$S^2 = 0$



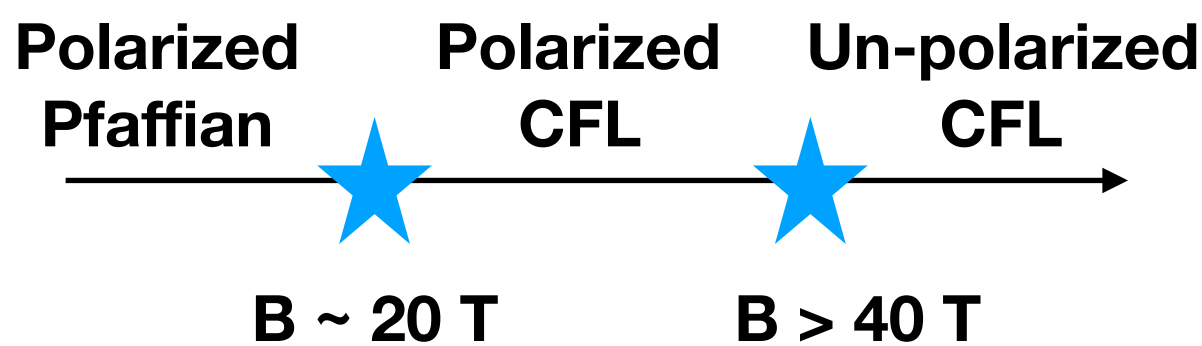
K=(1,1) K=(4,0) K=(4,4)

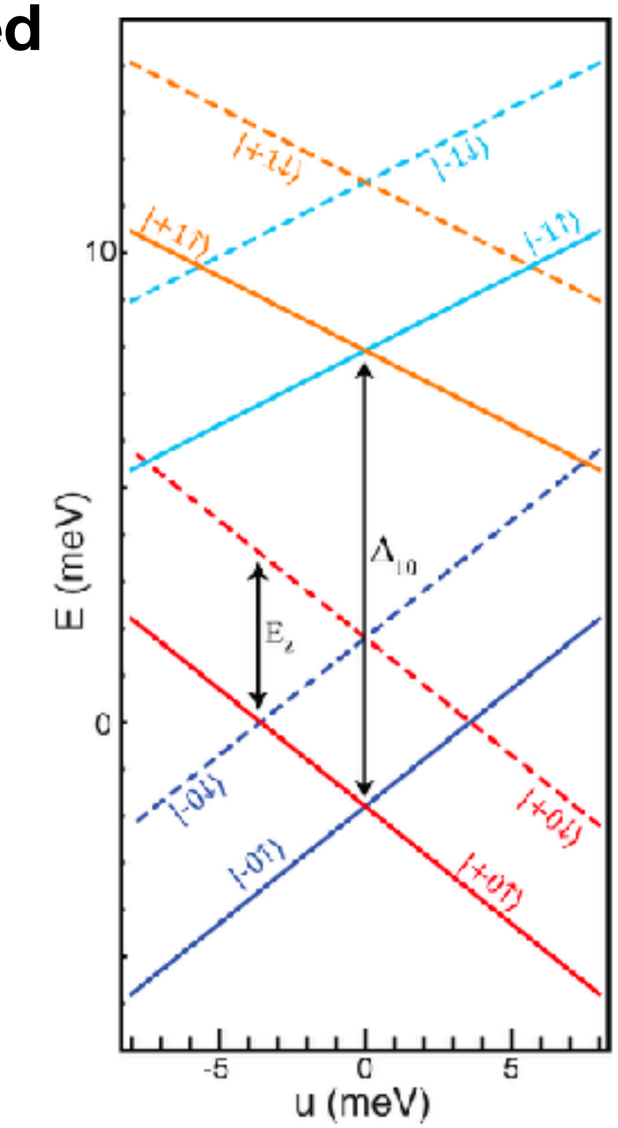
Clean Pfaffian to CFL in bilayer graphene



Continuously rotating the N=0 LL into a N=1LL with field

$$|\Theta,\sigma\rangle = \sin(\Theta)|0,B,\sigma\rangle + \cos(\Theta)|1,B'\sigma\rangle$$





It would be a Pfaffian with a twist:

- SU(2) skyrmions and ferro-magnet coexisting with Pfaffian physics

Summary

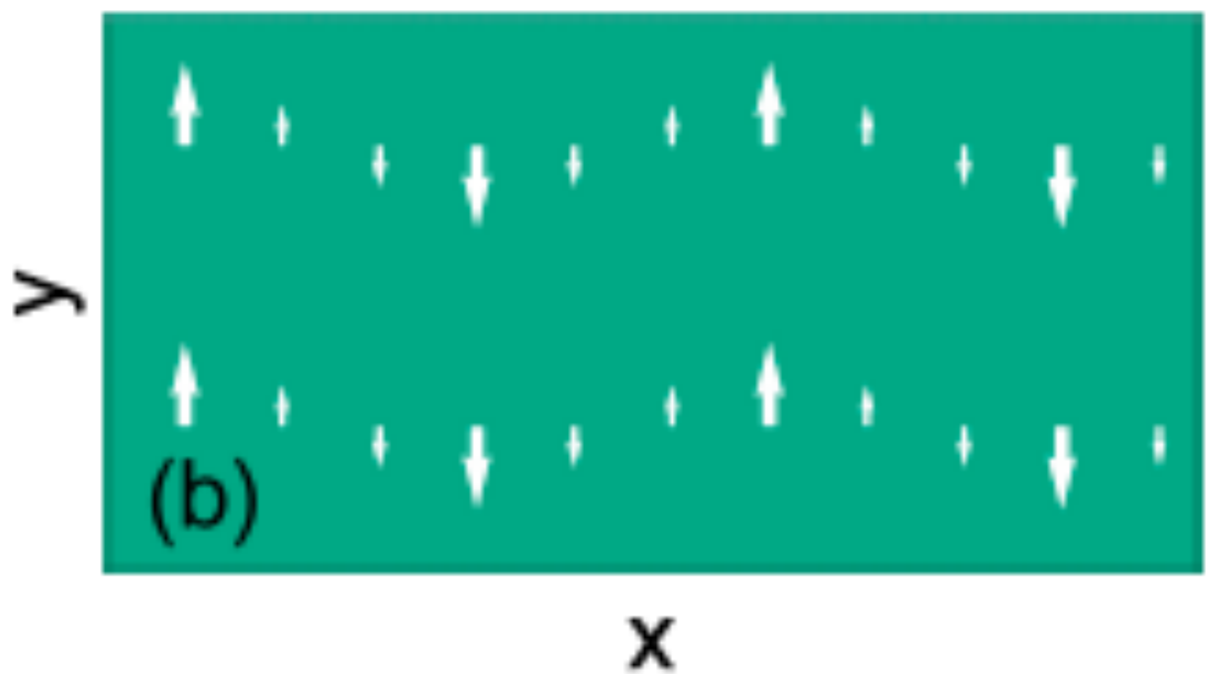
Alternative platforms for fractional quantum Hall physics like Graphene, ZnO, AlAs, allow to realize a wealth of phase transitions between fractionalized phases of matter.

- Bilayer graphene is a promising platform to realize the Quantum phase transition between Pfaffian and composite fermi liquid tuned by perpendicular field.

- Stoner transition of composite fermi liquids in AlAs (arXiv:1802.02167).

The shear sound of 2D Fermi liquids

Shear sound of interacting Fermi liquids



Jun Yong Khoo

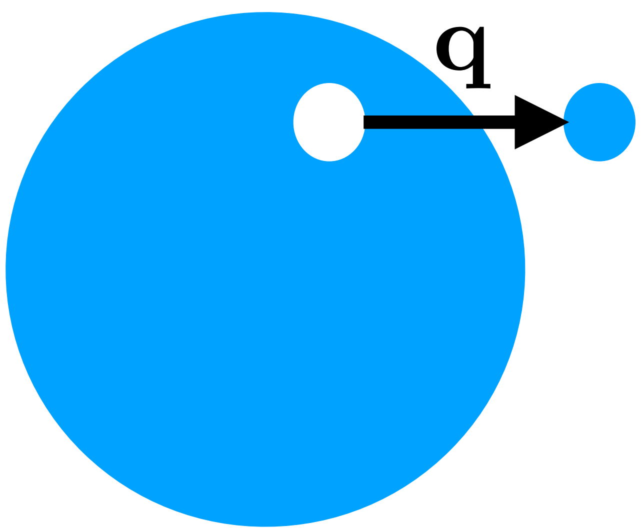
MIT

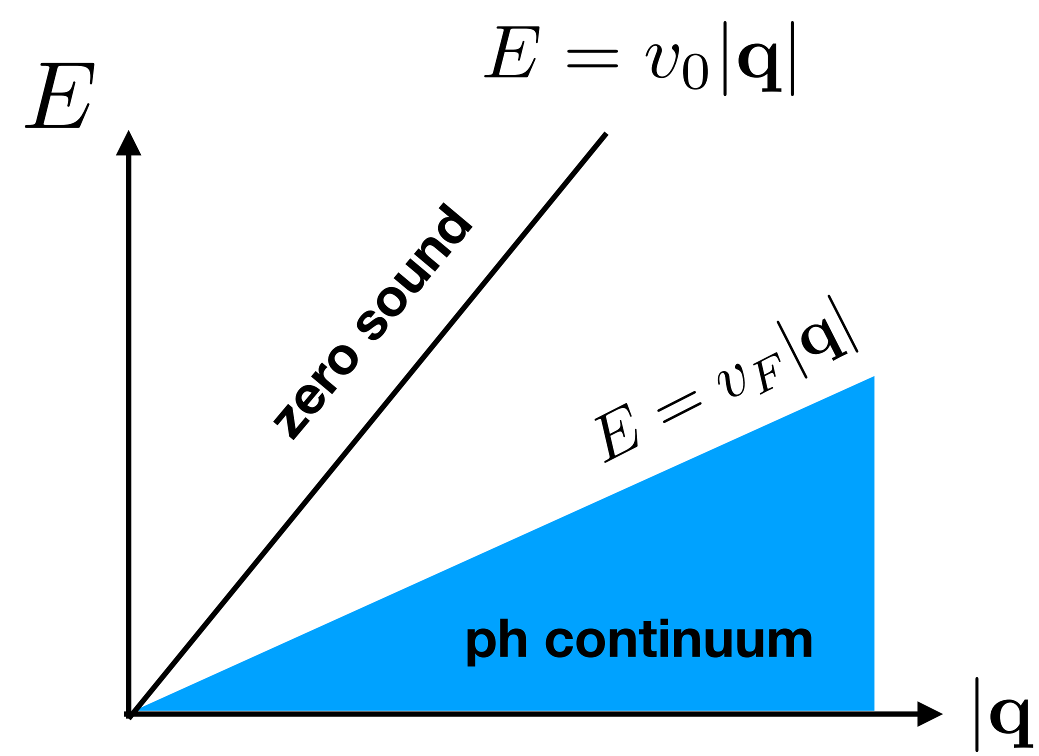
Should appear rather generically in 2D Fermi liquids when quasiparticles become twice as heavy

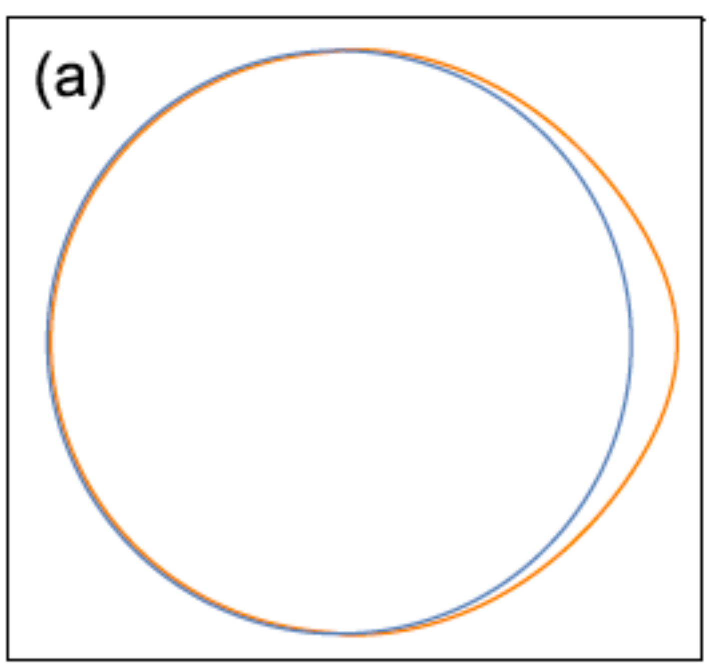
$$m^* > 2m_0$$

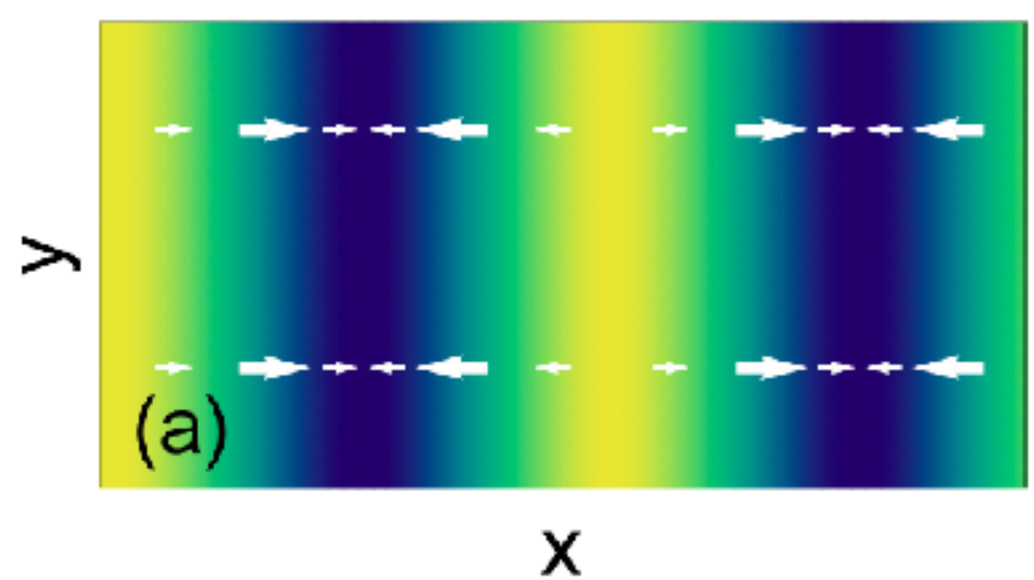
arXiv:1806.04157

Zero sound and particle-hole continuum



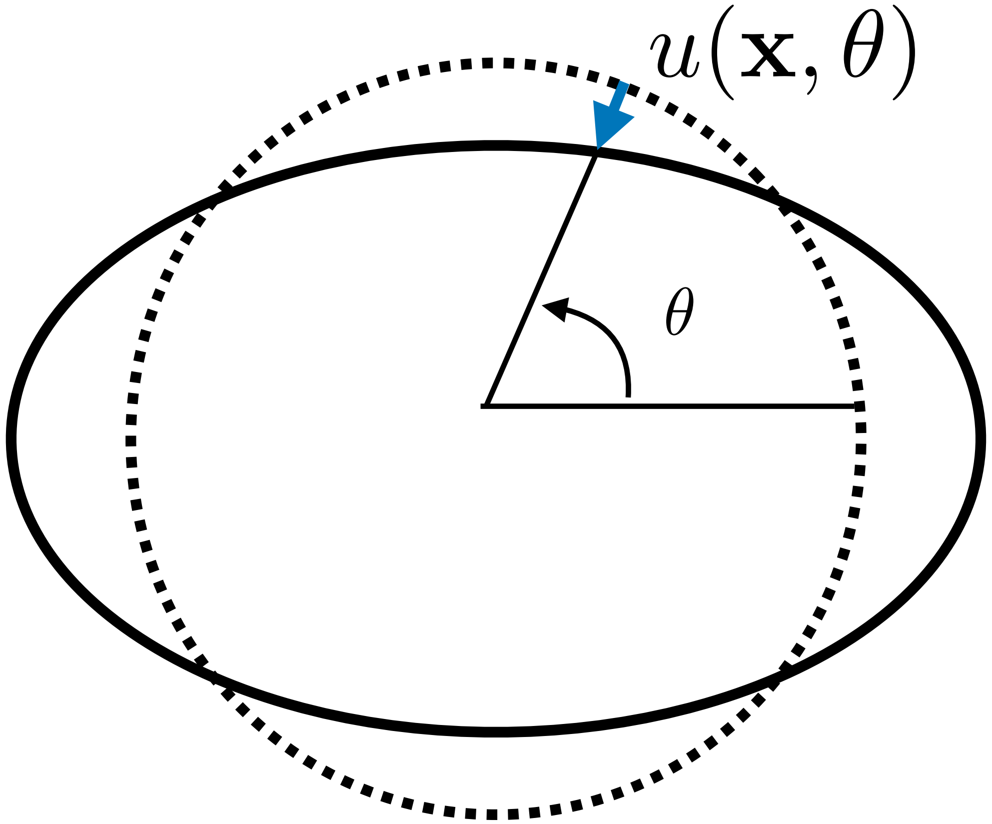






Landau Fermi liquid Theory or Bosonization of 2D Fermi liquids

2D Fermi liquids have an infinite number of slow variables



State is parametrized by space-time dependent Fermi radius

$$p_F(\mathbf{x}, \theta) = p_{F0} + u(\mathbf{x}, \theta)$$

Radius has commutation relations analogous to 1D

$$\partial_n = \hat{\mathbf{p}}_\theta \cdot \partial_{\mathbf{x}}$$

$$[\hat{u}_{\mathbf{x},\theta}, \hat{u}_{\mathbf{x}',\theta'}] = \frac{(2\pi)^2}{i p_{\mathrm{F}}} \delta(\theta - \theta') \partial_n \delta(\mathbf{x} - \mathbf{x}') + O(\hat{u})$$

A. Luther, Phys. Rev. B **19**, 320 (1979).

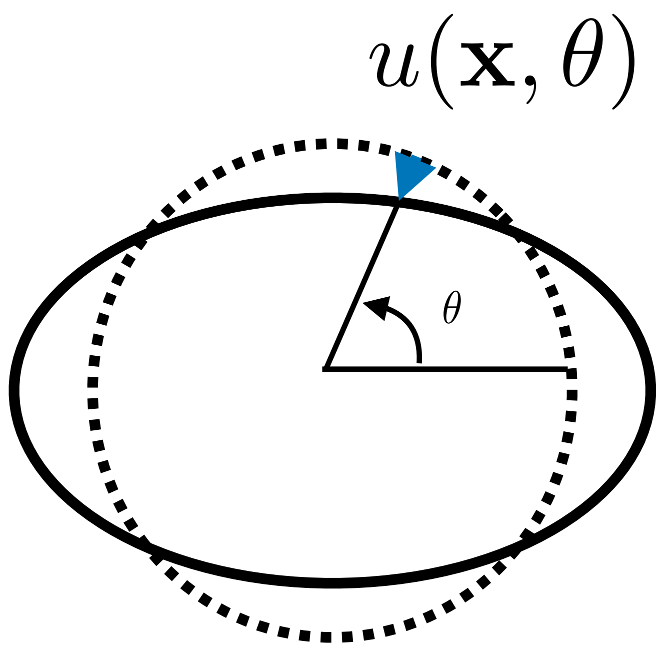
F. D. M. Haldane, eprint arXiv:cond-mat/0505529

A. H. Castro Neto and E. Fradkin, Phys. Rev. Lett. **72**, 1393 (1994).

D. F. Mross and T. Senthil, Phys. Rev. B **84**, 165126 (2011).

S. Golkar, D. X. Nguyen, M. M. Roberts, and D. T. Son, Phys. Rev. Lett. **117**, 216403 (2016).

Bosonization of 2D Fermi liquids



$$\partial_n = \hat{\mathbf{p}}_\theta \cdot \partial_{\mathbf{x}}$$

$$[\hat{u}_{\mathbf{x},\theta}, \hat{u}_{\mathbf{x}',\theta'}] = \frac{(2\pi)^2}{i p_{\mathrm{F}}} \delta(\theta - \theta') \partial_n \delta(\mathbf{x} - \mathbf{x}') + O(\hat{u})$$

$$\hat{H} = \int d^2\mathbf{x} \; \hat{u}^\dagger_{\mathbf{x},\theta} h_{\theta,\theta'} \hat{u}_{\mathbf{x},\theta'}$$

$$h_{\theta,\theta'} = \frac{v_F p_F}{2} \left(\delta(\theta' - \theta) + \frac{F(\theta' - \theta)}{2\pi} \right)$$

Landau function:

$$F(\theta' - \theta)$$

Quantum version of kinetic equation:

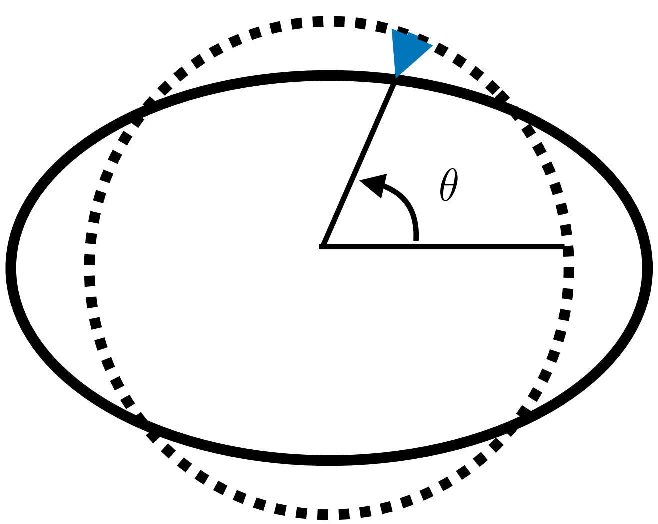
$$\hat{u}_{\mathbf{q},\theta} \equiv \int d^2\mathbf{x} \; \hat{u}_{\mathbf{x},\theta} e^{-i\mathbf{q}\cdot\mathbf{x}}$$

$$i\partial_t \hat{u}_{\mathbf{q},\theta} = \left[\hat{u}_{\mathbf{q},\theta}, \hat{H} \right] = K_{\theta,\theta'} \hat{u}_{\mathbf{q},\theta'},$$

$$K(\theta, \theta') = v_{\mathrm{F}} \mathbf{q} \cdot \hat{\mathbf{p}}_\theta \left(\delta(\theta - \theta') + \frac{1}{2\pi} F(\theta - \theta') \right)$$

$$v^\dagger_\theta G_{\theta,\theta'} w_{\theta'} \equiv \int \frac{d\theta d\theta'}{(2\pi)^2} v(\theta) G(\theta, \theta') w(\theta')$$

Mapping classical to quantum



Quantum version of kinetic equation:

$$i\partial_t \hat{u}_{\mathbf{q},\theta} = \left[\hat{u}_{\mathbf{q},\theta}, \hat{H}\right] = K_{\theta,\theta'}\hat{u}_{\mathbf{q},\theta'},$$

$$K(\theta,\theta') = v_{\mathrm{F}}\mathbf{q}\cdot\hat{\mathbf{p}}_\theta\left(\delta(\theta-\theta') + \frac{1}{2\pi}F(\theta-\theta')\right)$$

$$K = TK^\dagger T^{-1}, \;\; T_{\theta,\theta'} = \frac{(2\pi)^2\mathbf{q}\cdot\hat{\mathbf{p}}_\theta}{p_{\mathrm{F}}}\delta(\theta-\theta')$$

For every classical eigen-function of the kinetic equation:

$$K_{\theta,\theta'}\psi_{\lambda,\mathbf{q},\theta'} \;=\; E_\lambda\psi_{\lambda,\mathbf{q},\theta}$$

We can construct a quantum bosonic eigen-mode:

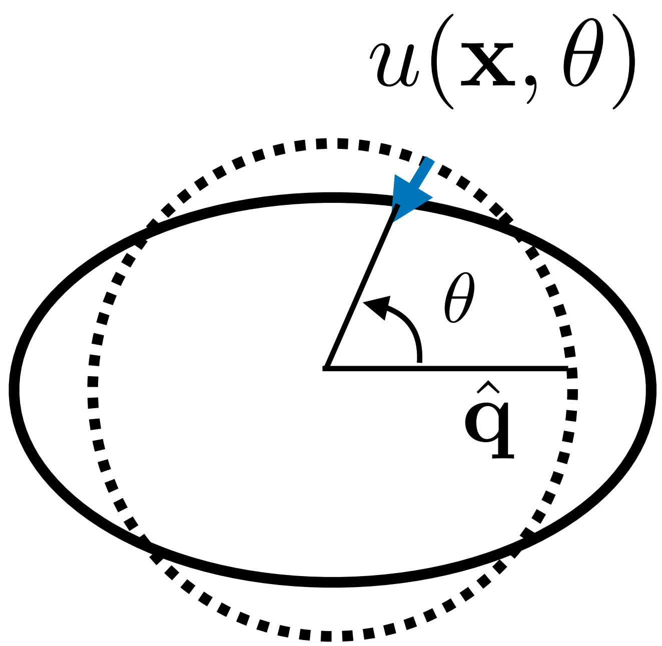
$$\hat{\psi}_{\lambda,\mathbf{q}} = \psi^\dagger_{\lambda,\mathbf{q},\theta}T^{-1}_{\theta,\theta'}\hat{u}_{\mathbf{q},\theta'}$$

Canonical normalisation:

$$\psi^\dagger_{\lambda,\mathbf{q},\theta}T^{-1}_{\theta,\theta'}\psi_{\lambda',\mathbf{q},\theta'} \;=\; \mathrm{sgn}(E_\lambda)\delta_{\lambda,\lambda'}$$

$$v^\dagger_\theta G_{\theta,\theta'}w_{\theta'} \equiv \int\frac{d\theta d\theta'}{(2\pi)^2}v(\theta)G(\theta,\theta')w(\theta')$$

Mapping classical to 1D tight binding



Mirror symmetry:

$$F(\theta) = F(-\theta), \; K_{\theta,\theta'} = K_{-\theta,-\theta'}$$

Even and Odd modes: $\sigma = \pm$

$$\psi^{\sigma}_{\lambda,\mathbf{q},\theta} = \sigma \psi^{\sigma}_{\lambda,\mathbf{q},-\theta}$$

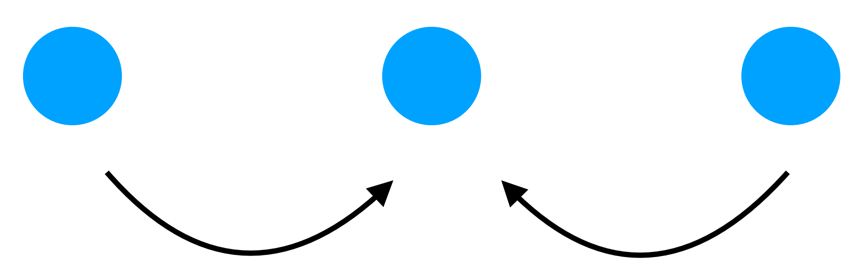
Angular momentum:

$$F(\theta) = F_0 + \sum_{l=1}^{\infty} 2F_l \cos(l\theta)$$

$$\psi^{+}_{\lambda,\theta} = \psi^{+}_{\lambda,0} + \sum_{l=1}^{\infty} 2\psi^{+}_{\lambda,l} \cos(l\theta),$$

$$\psi^{-}_{\lambda,\theta} = \sum_{l=1}^{\infty} 2\psi^{-}_{\lambda,l} \sin(l\theta).$$

$$E^{\sigma}_{\lambda} \psi^{\sigma}_{\lambda,l+1} = t_l \psi^{\sigma}_{\lambda,l} + t_{l+2} \psi^{\sigma}_{\lambda,l+2}$$



$$t_l = v_{\mathrm{F}} q(1 + F_l)/2$$

Mapping classical to 1D tight binding

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$$F(\theta) = F_0 + \sum_{l=1}^{\infty} 2F_l \cos(l\theta)$$

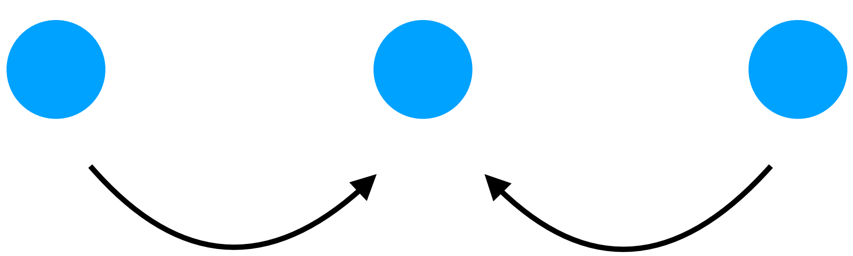
$$\psi^{+}_{\lambda,\theta} = \psi^{+}_{\lambda,0} + \sum_{l=1}^{\infty} 2\psi^{+}_{\lambda,l} \cos(l\theta),$$

$$\psi^{-}_{\lambda,\theta} = \sum_{l=1}^{\infty} 2\psi^{-}_{\lambda,l} \sin(l\theta).$$

Even and Odd modes: $\sigma = \pm$

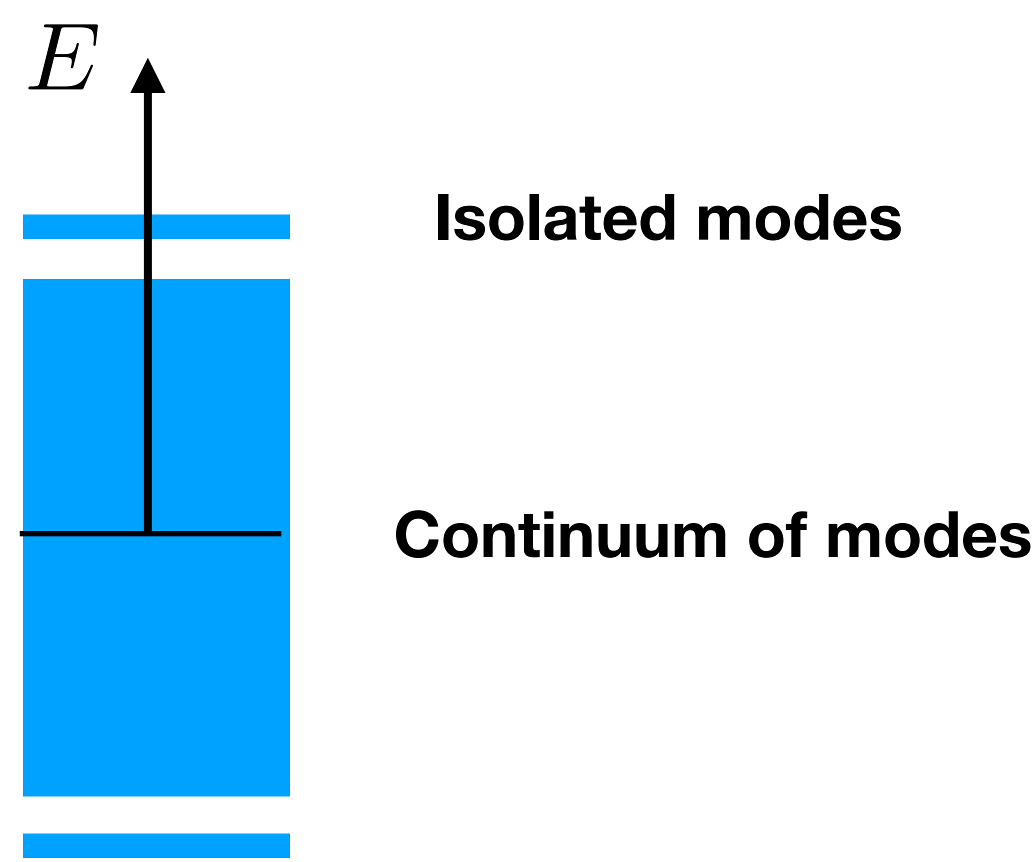
$$\psi^{\sigma}_{\lambda,\mathbf{q},\theta} = \sigma \psi^{\sigma}_{\lambda,\mathbf{q},-\theta}$$

$$E^{\sigma}_{\lambda} \psi^{\sigma}_{\lambda,l+1} = t_l \psi^{\sigma}_{\lambda,l} + t_{l+2} \psi^{\sigma}_{\lambda,l+2}$$



$$t_l = v_{\mathrm{F}} q (1 + F_l)/2$$

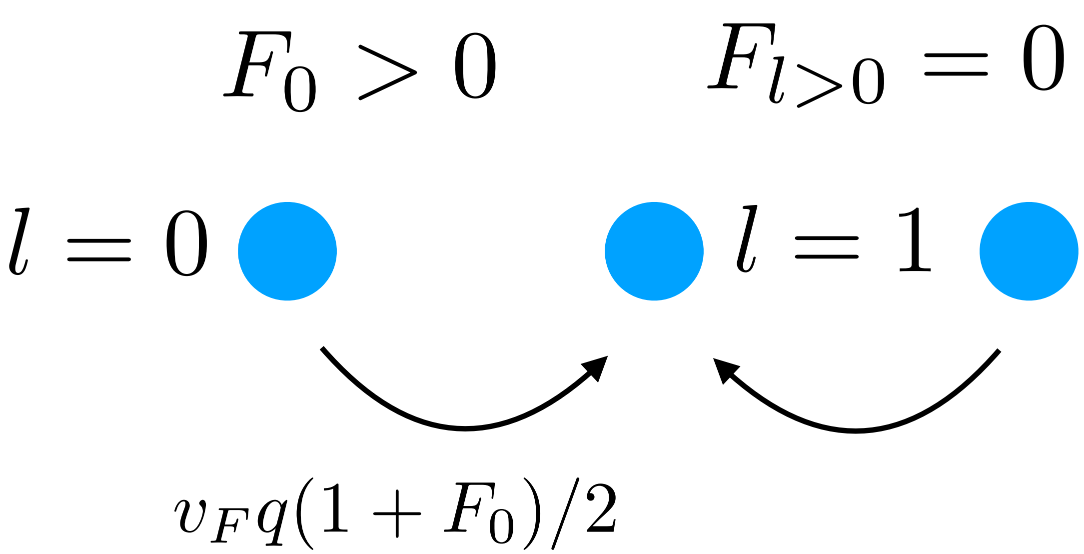
Landau parameters play role of bond disorder



A. Kumar and D. L. Maslov, Phys. Rev. B 95, 165140 (2017).

Another sound

Only one bond is defective:



$$E^{\sigma}_{\lambda}\psi^{\sigma}_{\lambda,l+1} = t_l\psi^{\sigma}_{\lambda,l} + t_{l+2}\psi^{\sigma}_{\lambda,l+2}$$

$$t_l = v_{\mathrm{F}} q(1 + F_l)/2$$

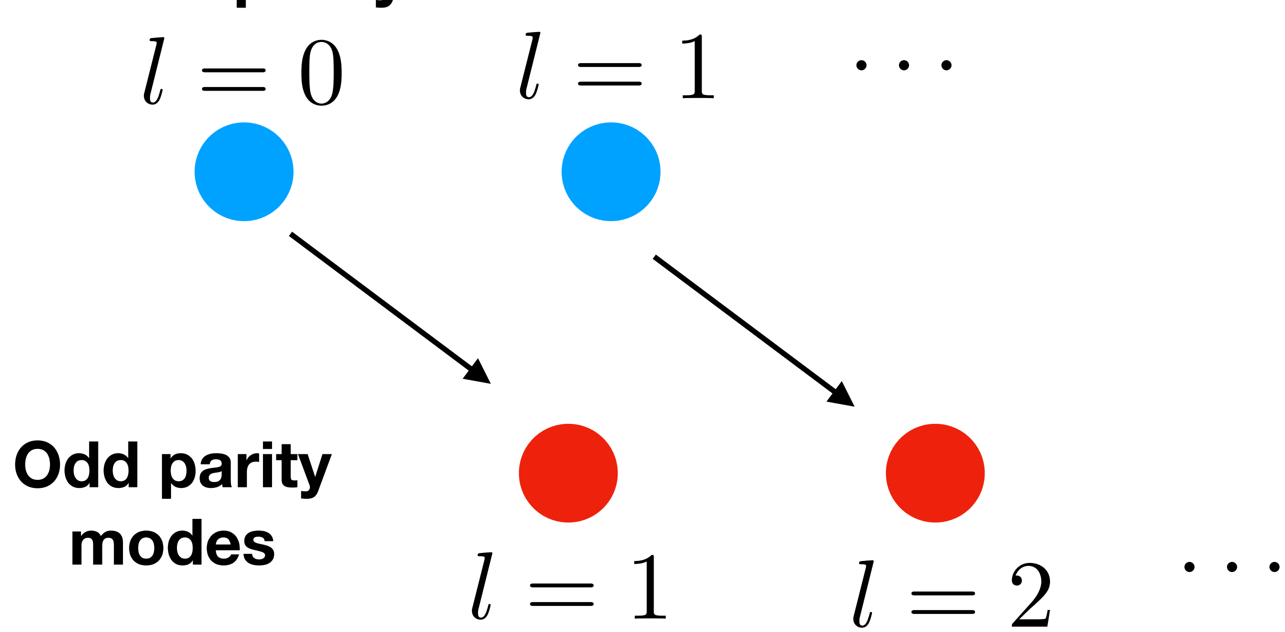
Mapping between even and odd sector problems

$$l \to l + 1$$

$$F'_{l+1} = F_l \quad \text{for } l \geq 1$$

$$F'_1 = 1 + 2F_0$$

Even parity modes



Implies existence of collective mode other than zero sound in model with non-zero:

$$\{F_0, F_1\}$$

Response functions

$$\hat{\mathcal{O}}_{\mathbf{q}} = \int d\theta O(\mathbf{q},\theta)\hat{u}_{\mathbf{q},\theta} = \sum_{\lambda} O_{\lambda,\mathbf{q}}\hat{\psi}_{\lambda,\mathbf{q}}$$

$$\rho_{\lambda,\mathbf{q}} = \mathrm{sgn}(E_\lambda)\frac{p_{\mathrm{F}}}{2\pi}\psi^{+}_{\lambda,0}$$

$$\mathbf{j}_{\lambda,\mathbf{q}} = \mathrm{sgn}(E_\lambda)\frac{v_{0\mathrm{F}}p_{\mathrm{F}}}{2\pi}(\psi^{+}_{\lambda,1}\hat{\mathbf{q}} + \psi^{-}_{\lambda,1}\hat{\mathbf{q}}_{\perp})$$

	Density	Longitudinal Current
Even Modes	$l=0$	$l=1$

	Transverse Current
Odd modes	$l=1$

Sharp peak in the transverse current-current correlation function with spectral weight:

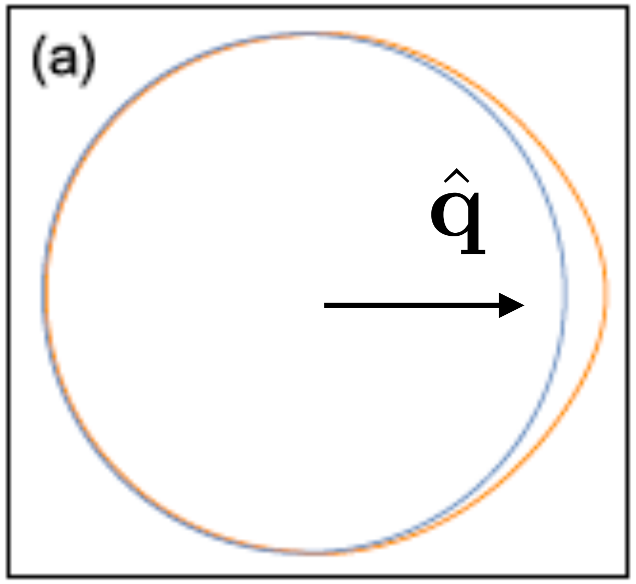
$$w_{j_\perp j_\perp} = \frac{p_{\mathrm{F}} q v_{0\mathrm{F}}^2}{16}\frac{F_1 - 1}{F_1^{3/2}} \qquad \text{vanishes as } F_1 \to 1$$

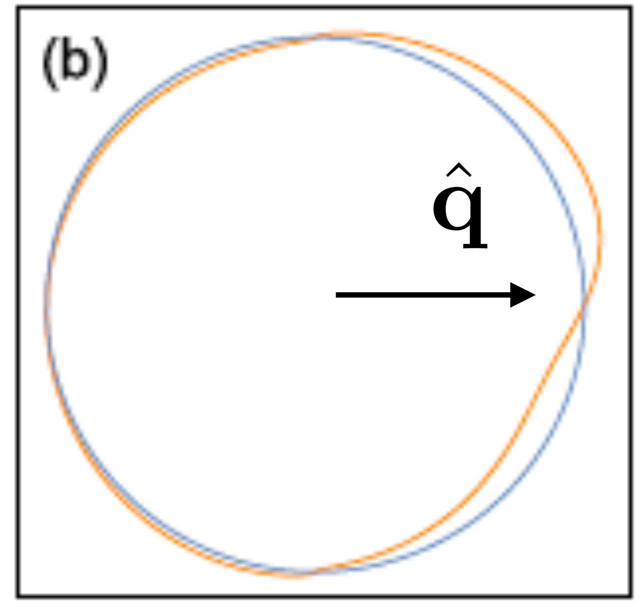
$$\mathrm{Im}\chi_{j_\perp j_\perp}(\mathbf{q},\omega) = -\mathcal{A}\frac{\pi v_{0\mathrm{F}}^2 p_{\mathrm{F}}^2}{(2\pi)^2}\sum_i \left|\psi^{-}_{i,1}\right|^2 \mathrm{sgn}(E_i)\delta(\omega - E_i^{-})$$

Shear vs zero sound

$$v_{zero} = \frac{1+F_0}{\sqrt{1+2F_0}}$$

$F_0 > 0$



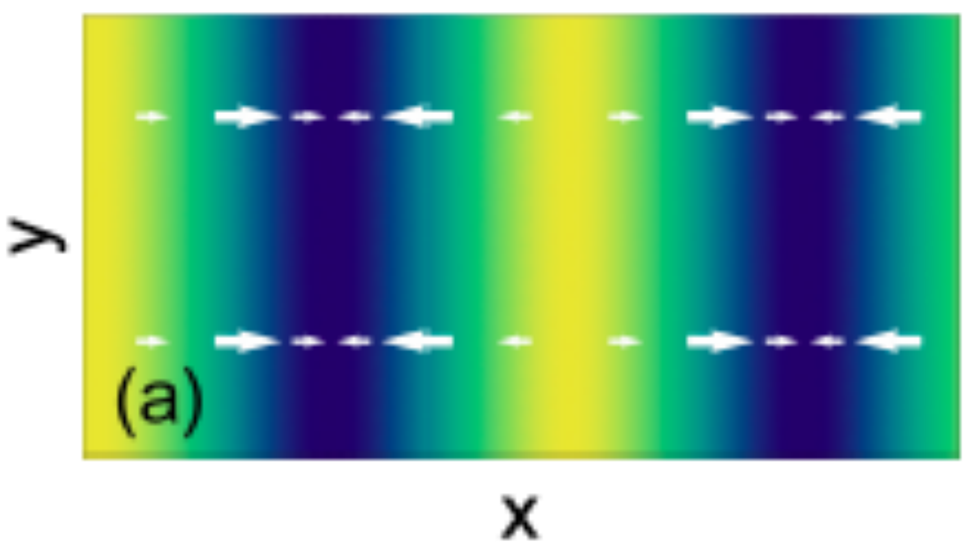


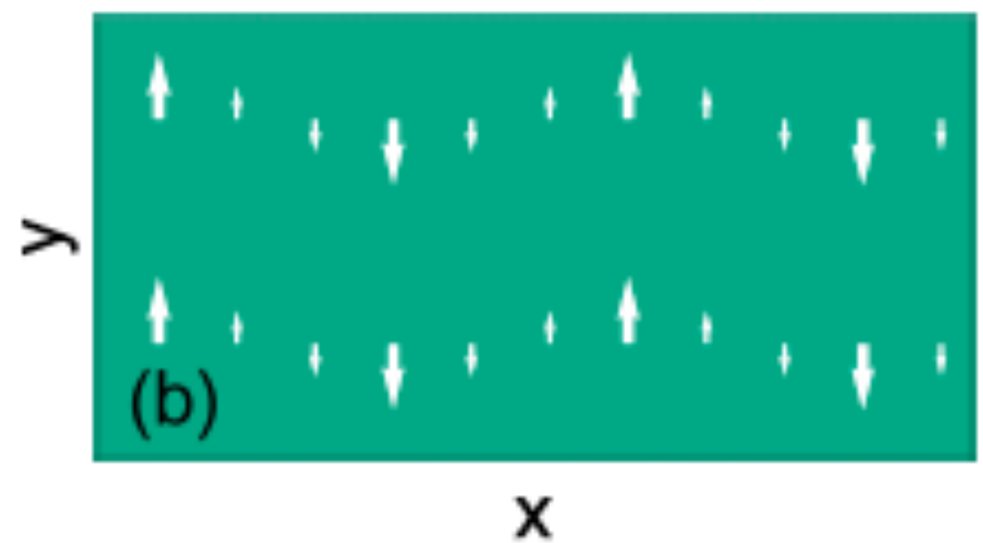
$$v_{shear} = \frac{1+F_1}{2\sqrt{F_1}}$$

$F_1 > 1$

Zero sound is longitudinal

Shear sound is purely transverse





$$\mathbf{j} \parallel \mathbf{q}$$

$$\mathbf{j} \perp \mathbf{q}$$

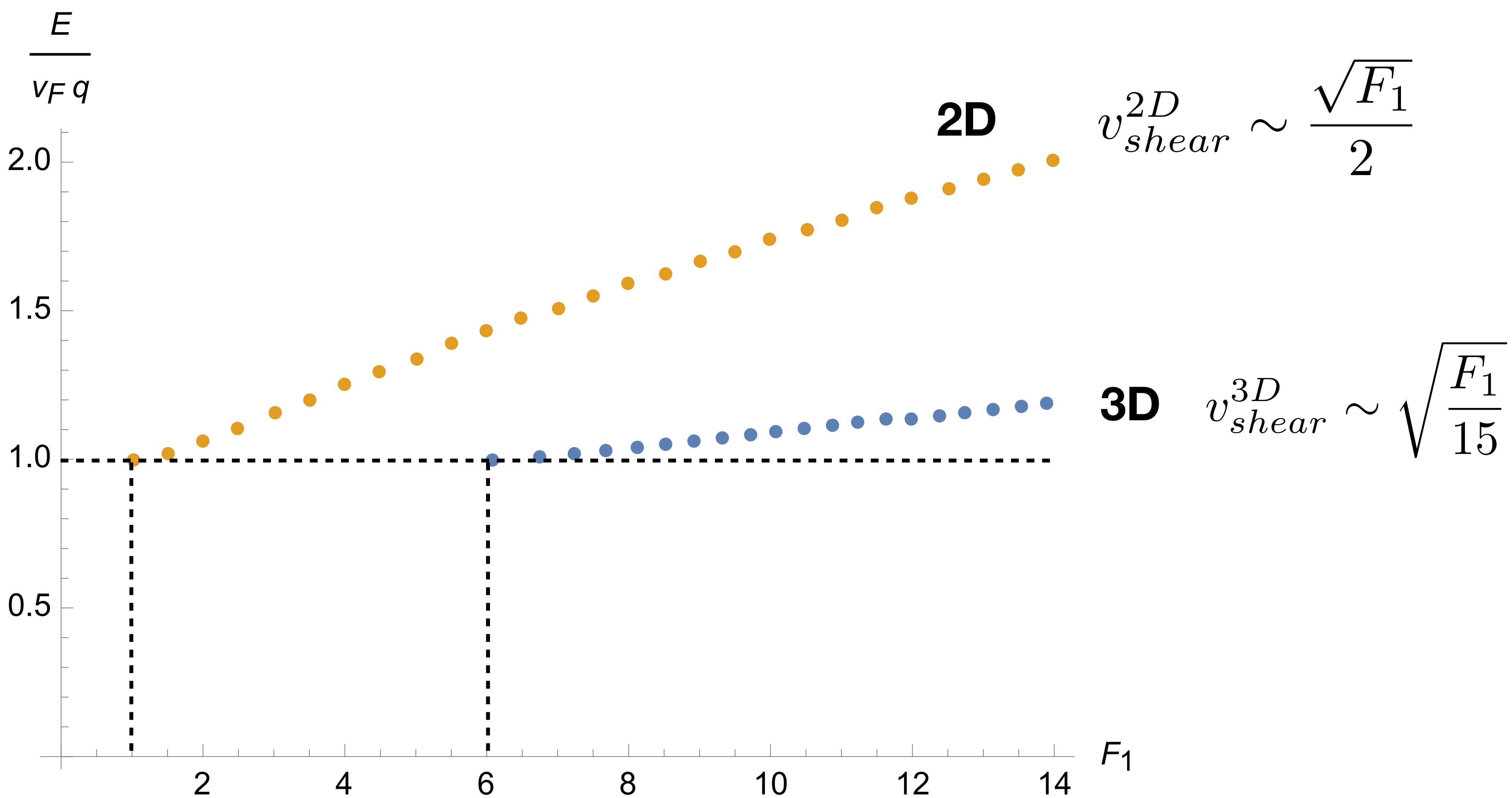
In metals zero sound is transformed into plasma mode

$$E \propto \sqrt{|\mathbf{q}|}$$

Shear sound remains linearly dispersing in metals

$$E = v_\perp |\mathbf{q}|$$

Shear sound 2D vs 3D



It should be easier in 2D fermi liquids. It should be present in metals as well.

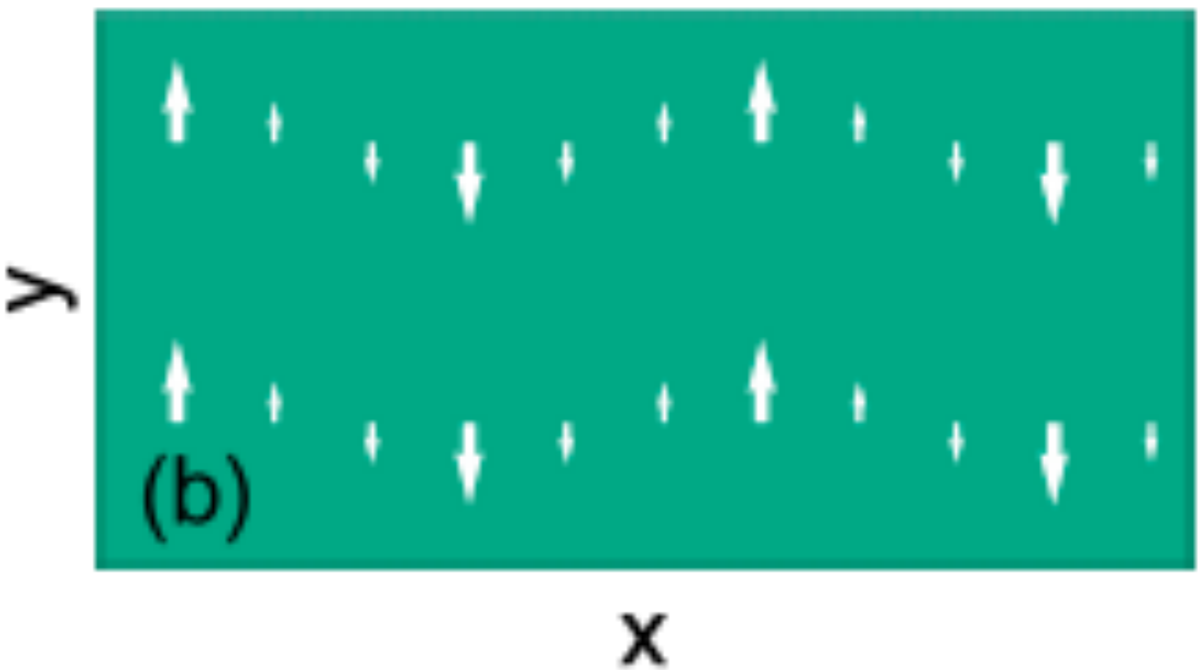
Shear sound was attempted to be measured in 3D Helium in 70's but results remained controversial (too close to ph continuum):

P. R. Roach and J. B. Ketterson, Phys. Rev. Lett. **36**, 736 (1976).

E. G. Flowers, R. W. Richardson, and S. J. Williamson, Phys. Rev. Lett. **37**, 309 (1976).

Shear sound and mass renormalization

Mode is Landau damped in weakly interacting Fermi liquids



Mode is expected to appear out of continuum when:

$$F_1 > 1$$

Mode is expected to appear when quasiparticles become twice as heavy:

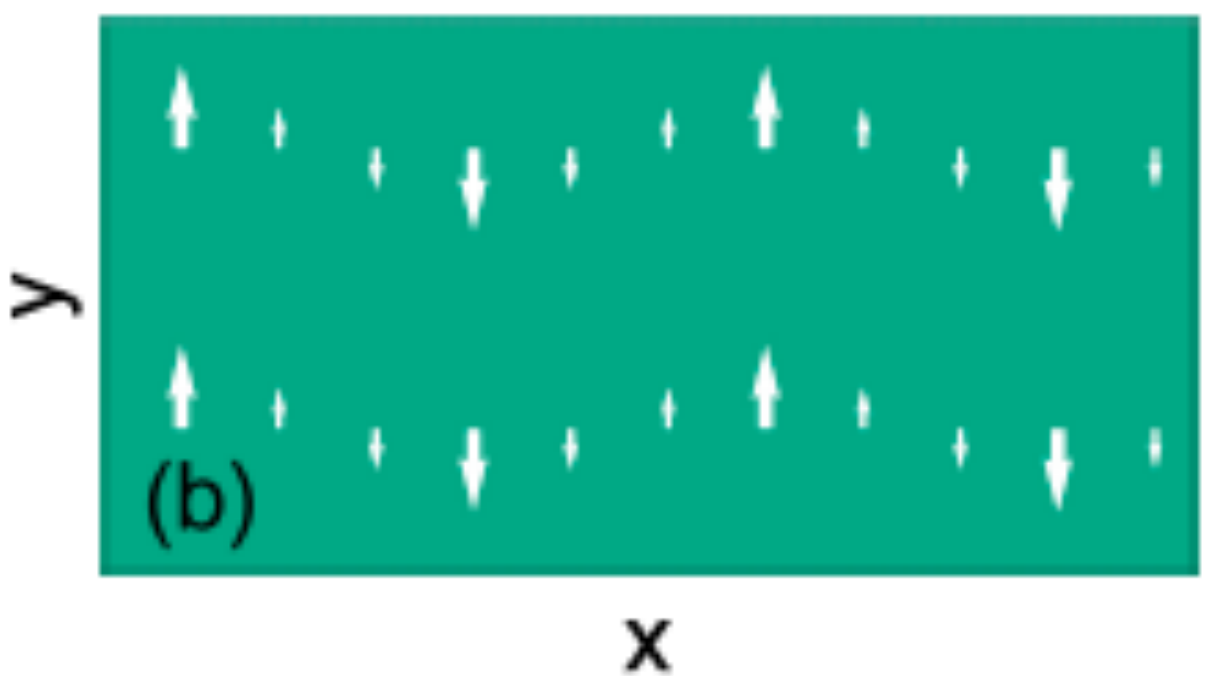
$$\frac{v_{F0}}{v_F} = \frac{m^*}{m_0} = 1 + F_1$$

Large variety of systems with mass enhancement near critical points could host undamped shear sound:

- He3 adsorbed on graphite — M. Neumann, J. Nyéki, B. Cowan, and J. Saunders, Science **317**, 1356 (2007).
- Overdoped metal in iron based superconductors (cuprates?) — Hashimoto et al. Science **336**, 1554 (2012).
- Quasi-2D heavy fermion materials (e.g. CeCoIn5) — Settai et al. Journal of Physics: Condensed Matter 13, L627 (2001)

Summary

Mode is Landau damped in weakly interacting Fermi liquids



Mode is expected to appear out of continuum when:

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Mode is expected to appear when quasiparticles become twice as heavy:

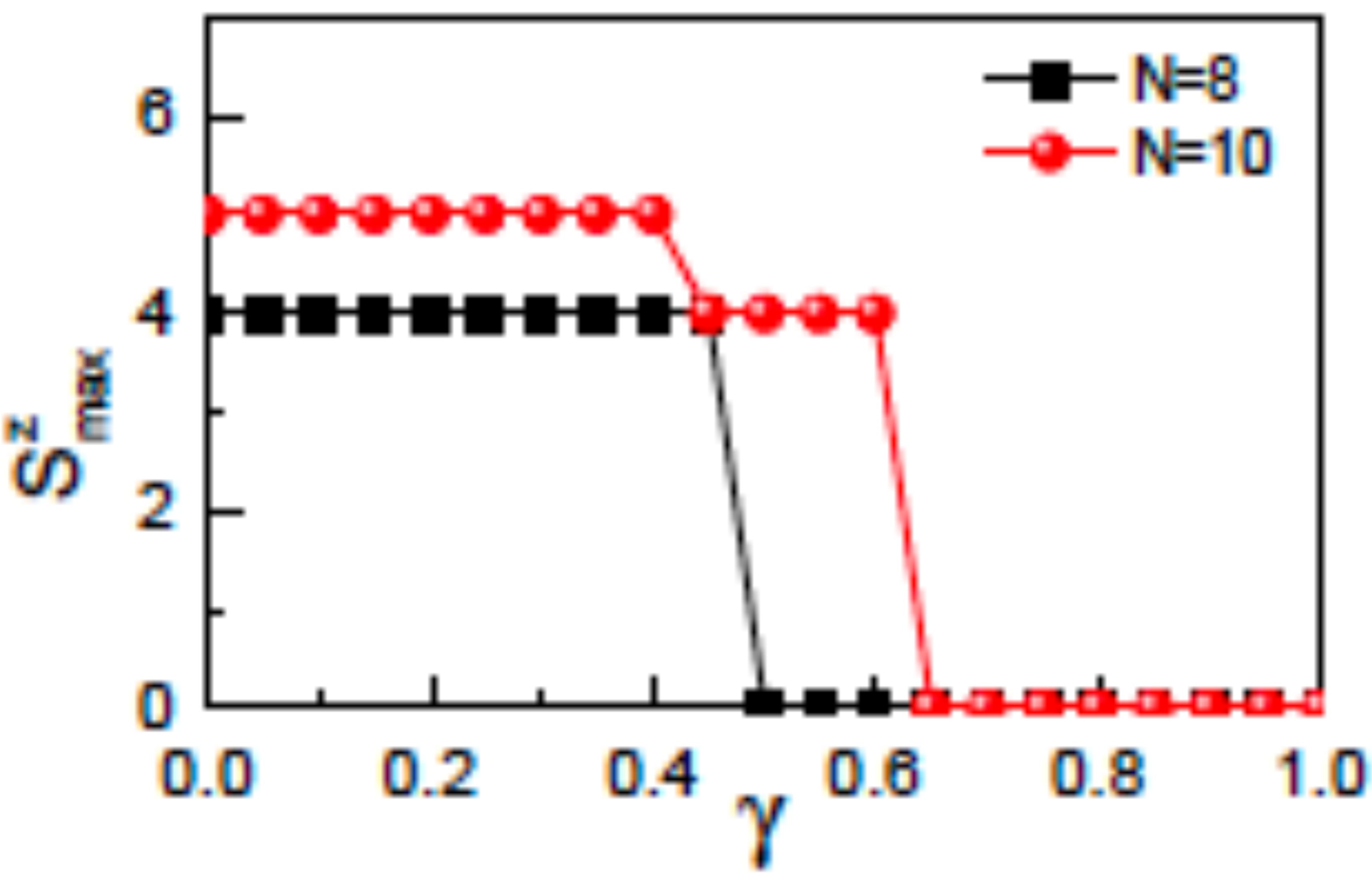
$$\frac{v_{F0}}{v_F} = \frac{m^*}{m_0} = 1 + F_1$$

Large variety of systems with mass enhancement near critical points could host undamped shear sound:

- He3 adsorbed on graphite surface
- Over-doped metal in iron based superconductors (cuprates?)
- Quasi-2D heavy fermion materials (e.g. CeCoIn5)

arXiv:1806.04157

Stoner



The composite Fermi liquid

Consider spin-full electrons at $\nu = \frac{1}{2}$

Slave-boson

Spinfull
neutral fermion

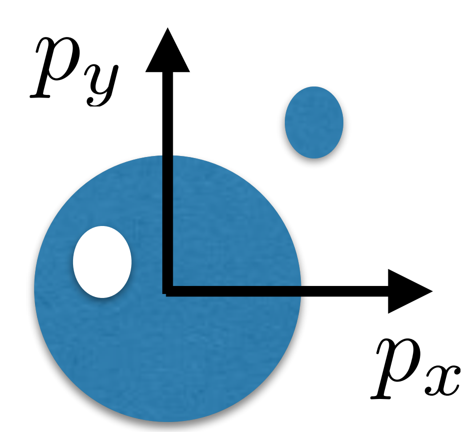
Electron $\longrightarrow$ $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ $\longleftarrow$ Spinless
charged boson

Boson forms Laughlin state:

$$\nu_b = \nu_e = \frac{1}{2}$$

$$\Psi_b = \prod_{i<j} (z_i - z_j)^2 e^{-\frac{|z_i|^2}{4l^2}}$$

Fermion forms fermi sea:



composite fermion fermi surface

The composite Fermi liquid

Consider spin-full electrons at $\nu = \frac{1}{2}$

Slave-boson

Spinfull neutral fermion

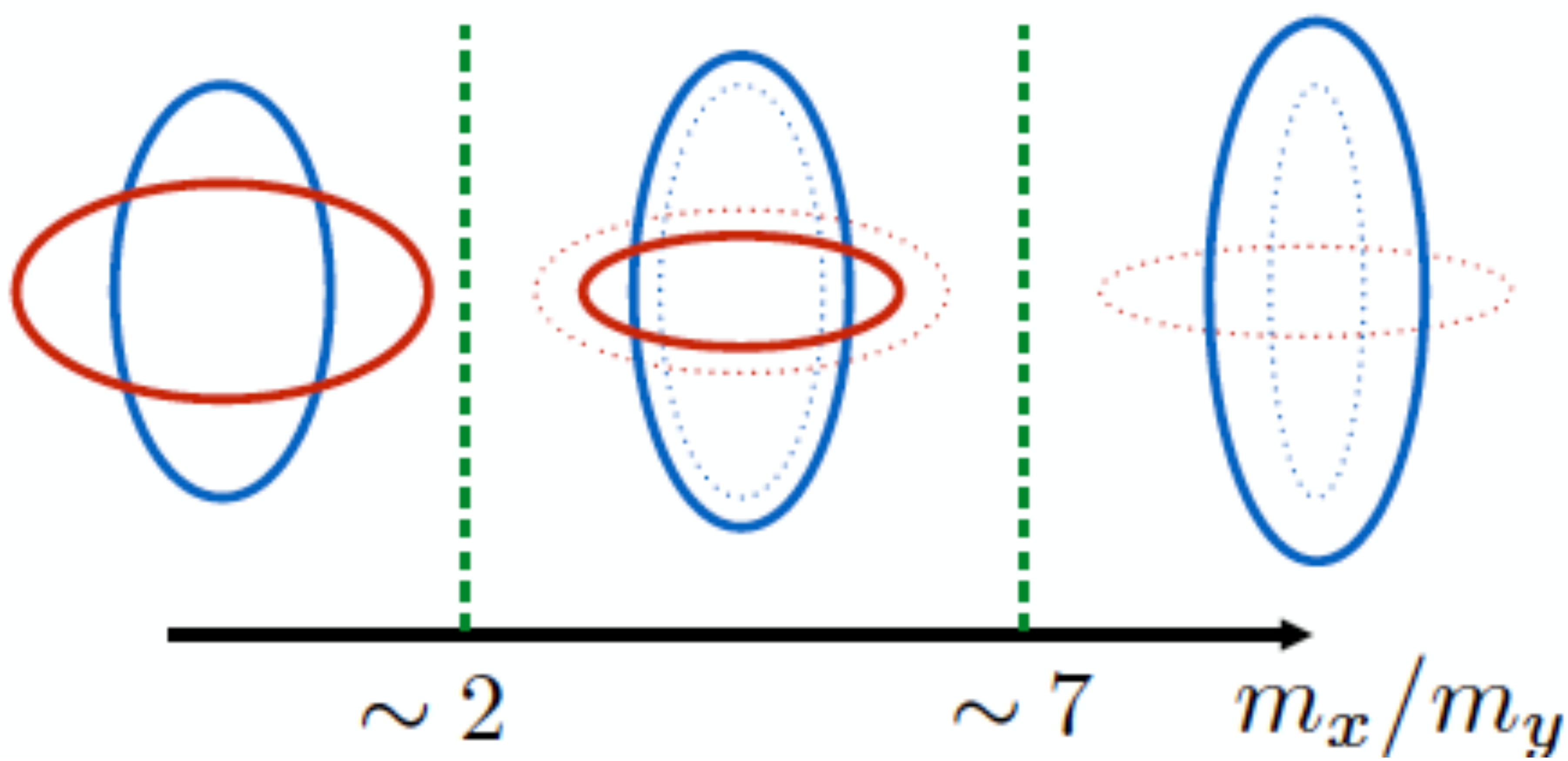
Electron $\longrightarrow$ $c^\dagger_{\sigma r} = f^\dagger_{\sigma r} b^\dagger_r$ $\longleftarrow$ Spinless charged boson

$$\nu_b = \nu_e = \frac{1}{2}$$

State of b	Mott - CDW	Laughlin state
Phase of electrons	U(1) spinon Fermi surface	Composite Fermi liquid
Name of f	Spinon	Composite fermion

Ising Stoner Instability of Composite Fermion Metal

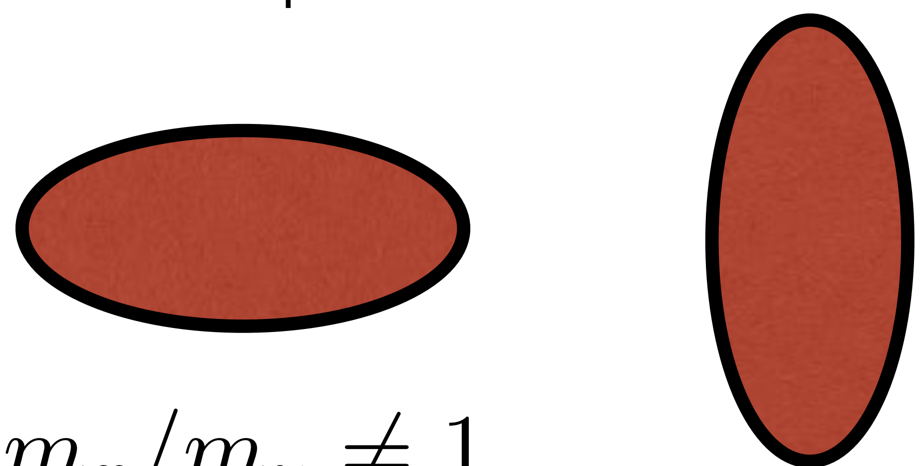
- Two components with rotated mass tensors rotated by $\pi/2$ undergo a an analogue of the Stoner transition:



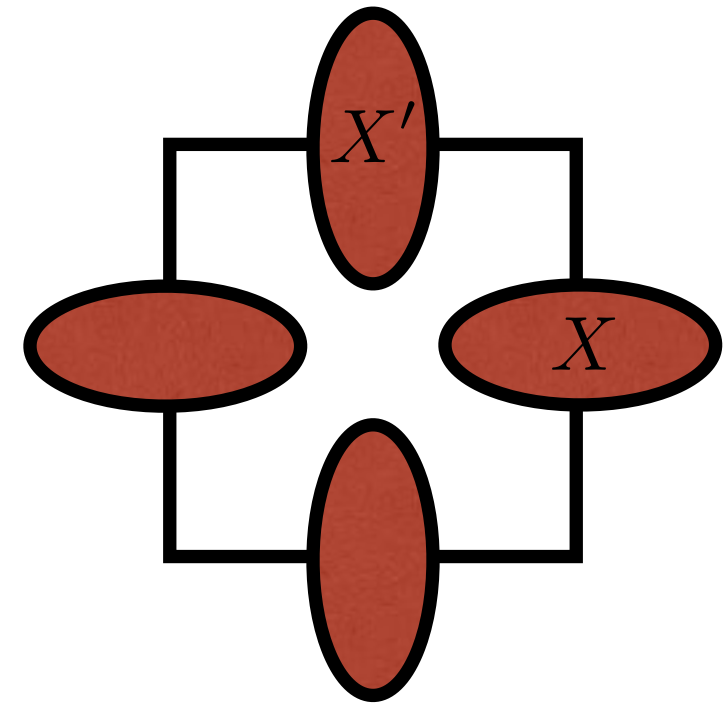
- Aluminum Arsenide $m_x/m_y \approx 5$

The Hamiltonian of our study

Aluminum Arsenide: Two valleys with anisotropic mass

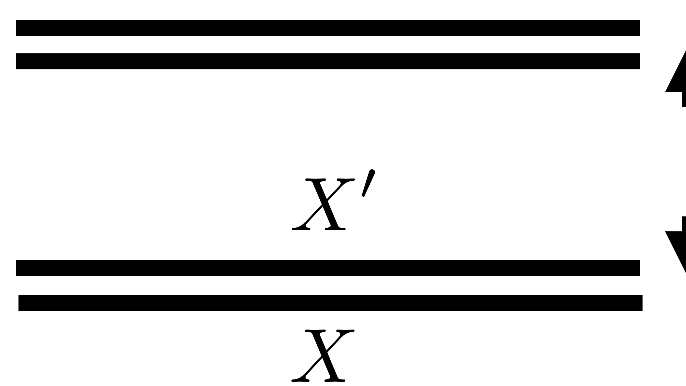


$$m_x/m_y \neq 1$$



$$\mathbf{p} \rightarrow \mathbf{p} + \mathbf{A}(\mathbf{r})$$

$$N = 0LL$$



$$\omega_c = \frac{B}{\sqrt{m_x m_y}}$$

Landau level projection endows electrons with a “shape”

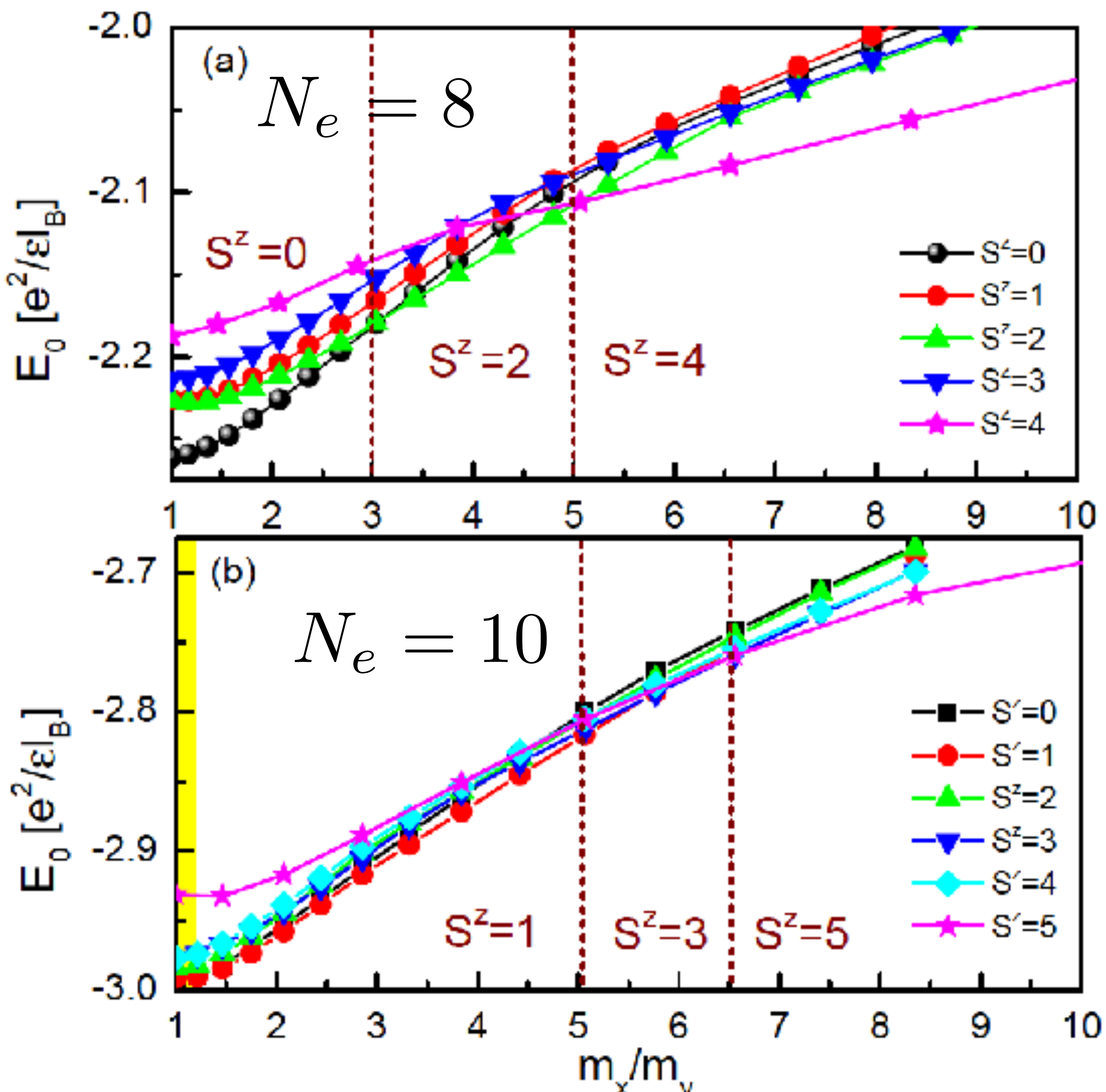
LL projection

$$v_0(q) = \frac{2\pi e^2}{\epsilon q} \quad \Rightarrow \quad v_{eff}(q) = \frac{2\pi e^2}{\epsilon q} F_i(q) F_j^*(q)$$

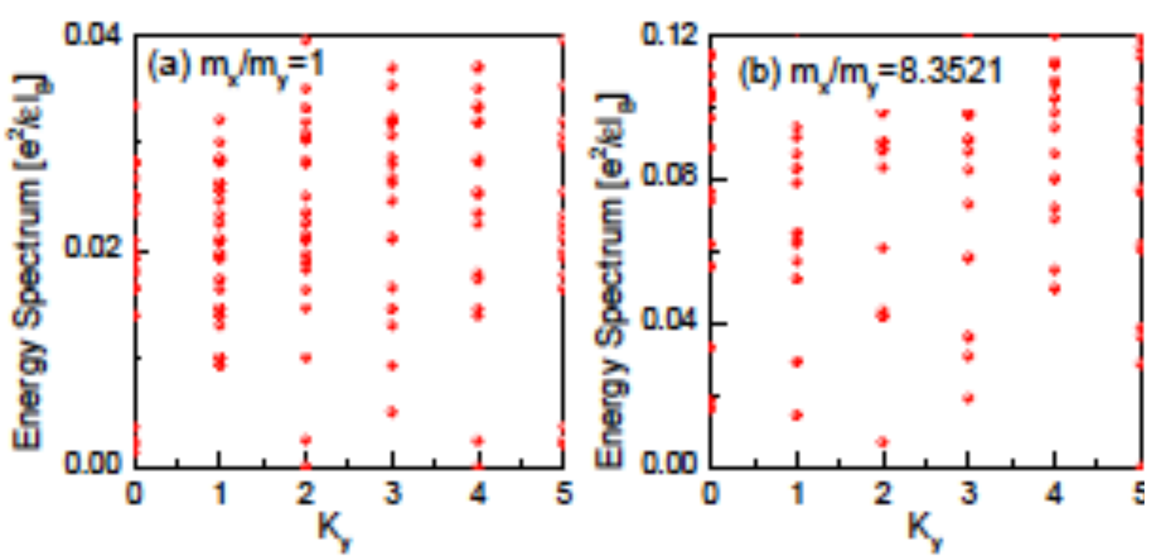
Ground state polarization

$$S^z = \frac{N_1 - N_2}{2}$$

Exact diagonalization on torus



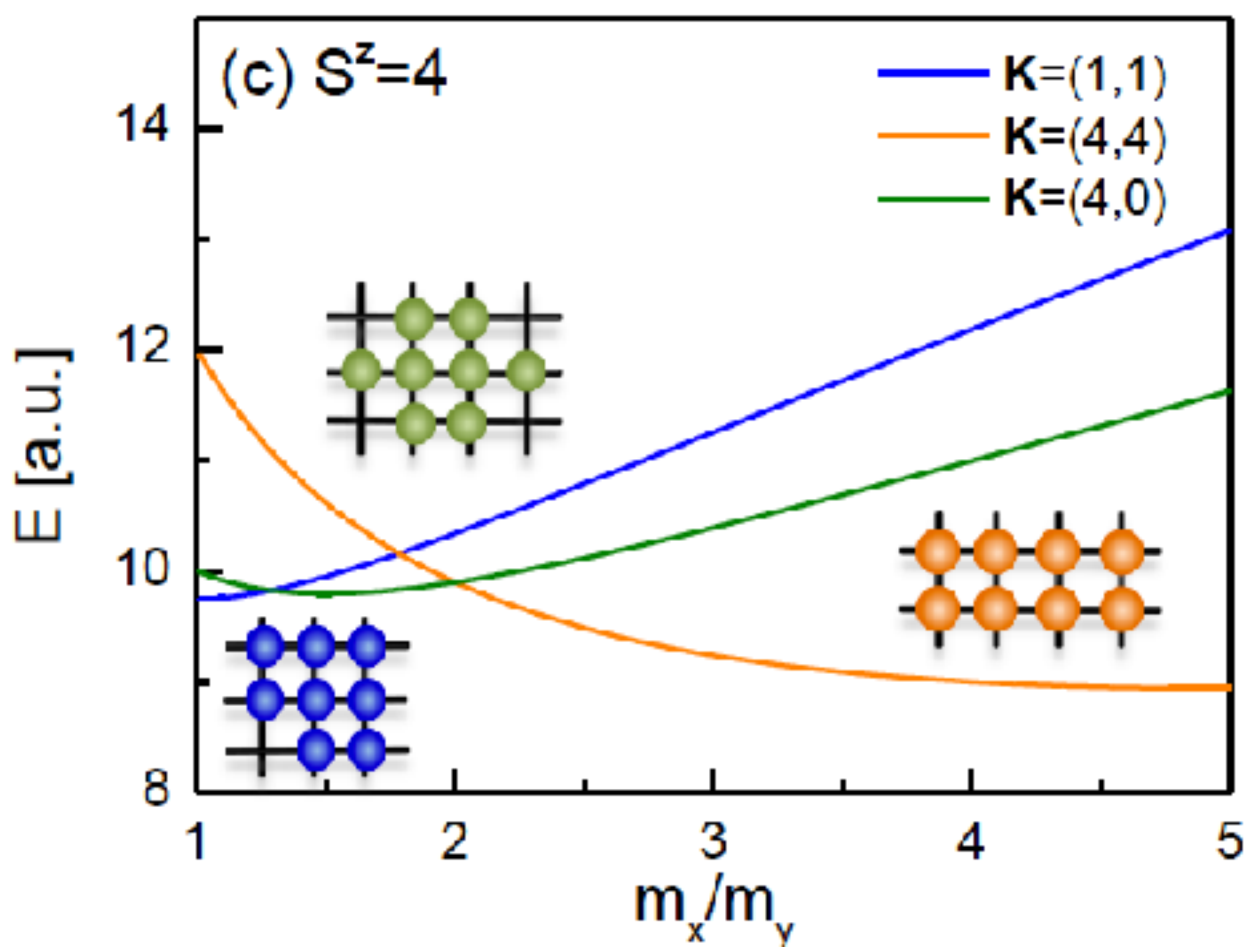
No clear gap in spectrum



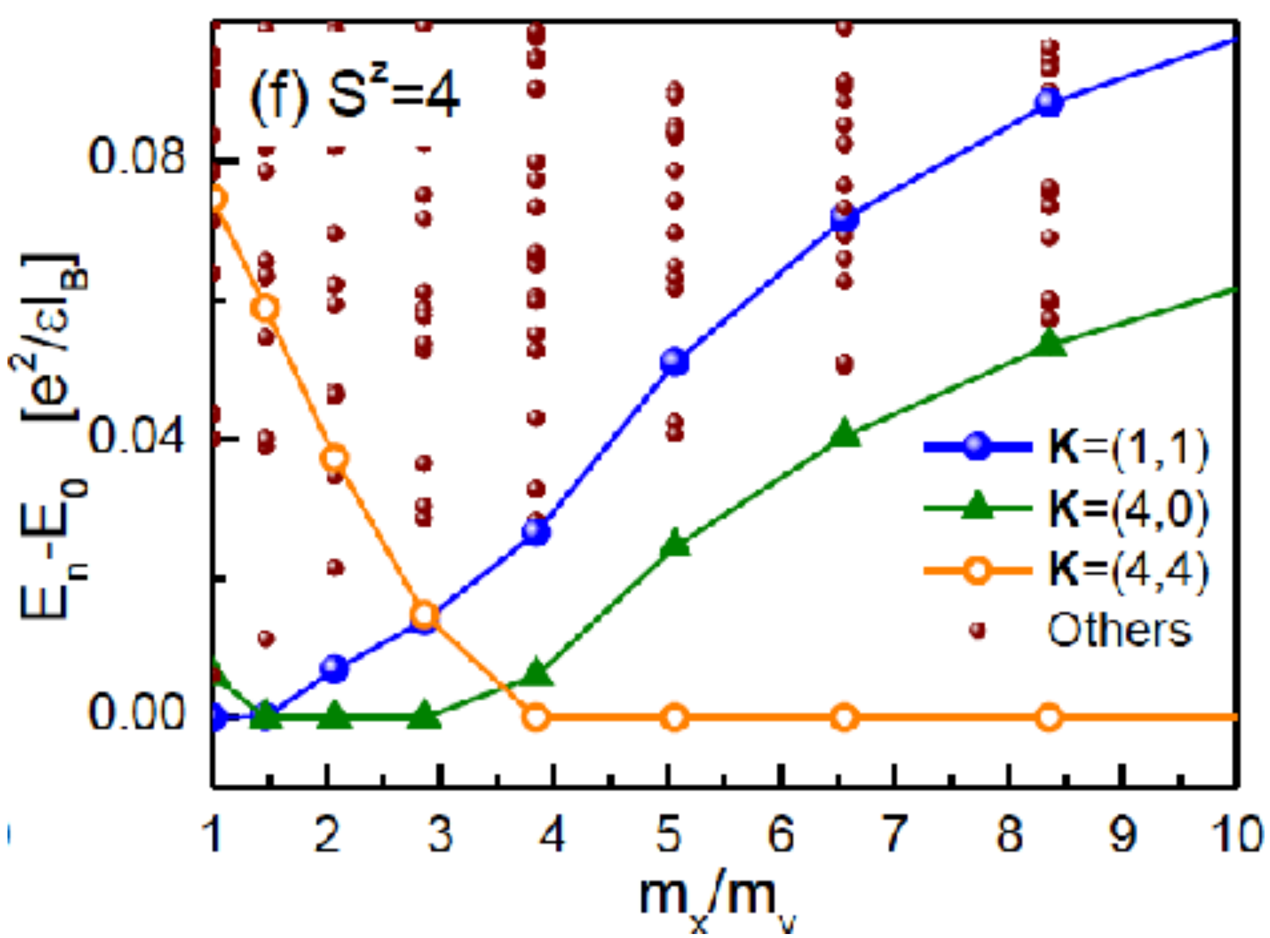
Numerics vs simple model

$$|\Psi_{\rm CFL}(\{\mathbf{d}_{\mathbf{i}}^{\uparrow},\mathbf{d}_{\mathbf{i}}^{\downarrow}\})\rangle = \det(\hat{t}_j(\mathbf{d}_{\mathbf{i}}^{\uparrow}))\det(\hat{t}_j(\mathbf{d}_{\mathbf{i}}^{\downarrow}))|\Phi_{1/2}^{\rm Bose}\rangle$$

Trial wave function

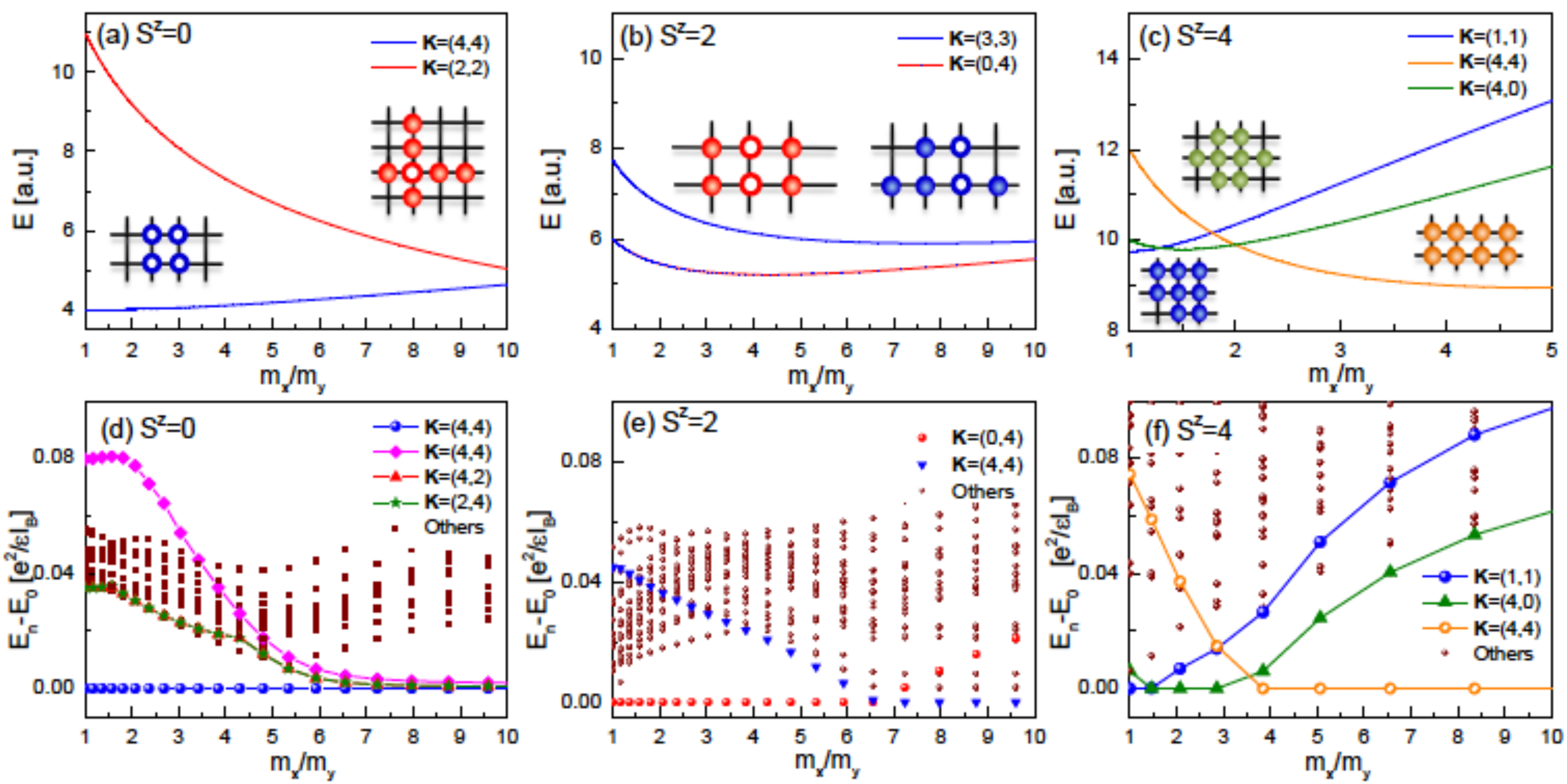


Exact diagonalization



Numerics vs simple model

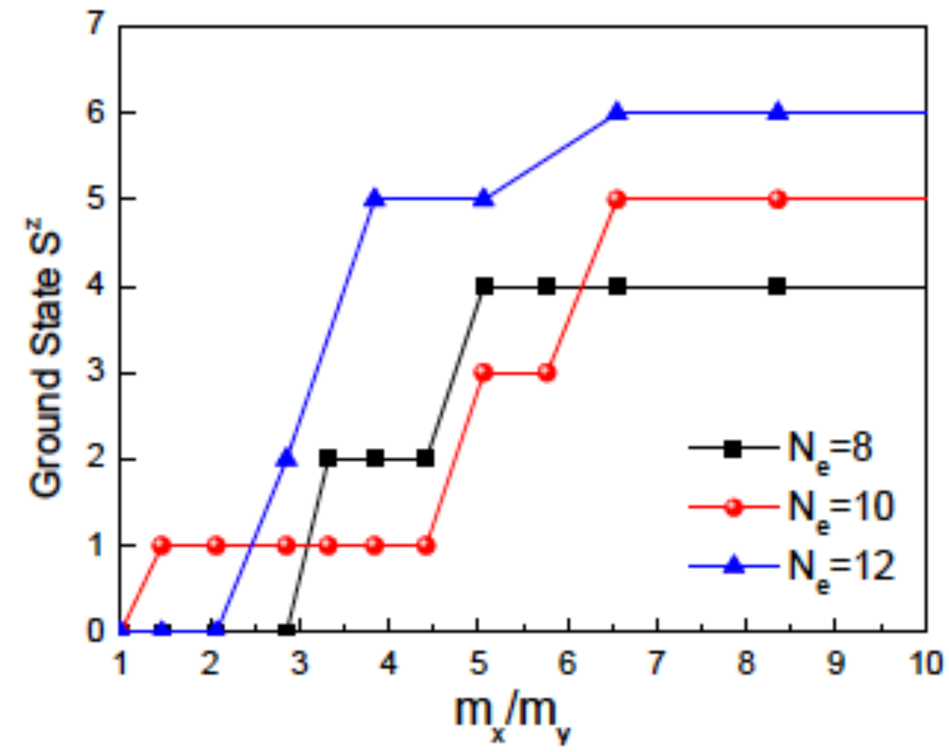
Trial wave function



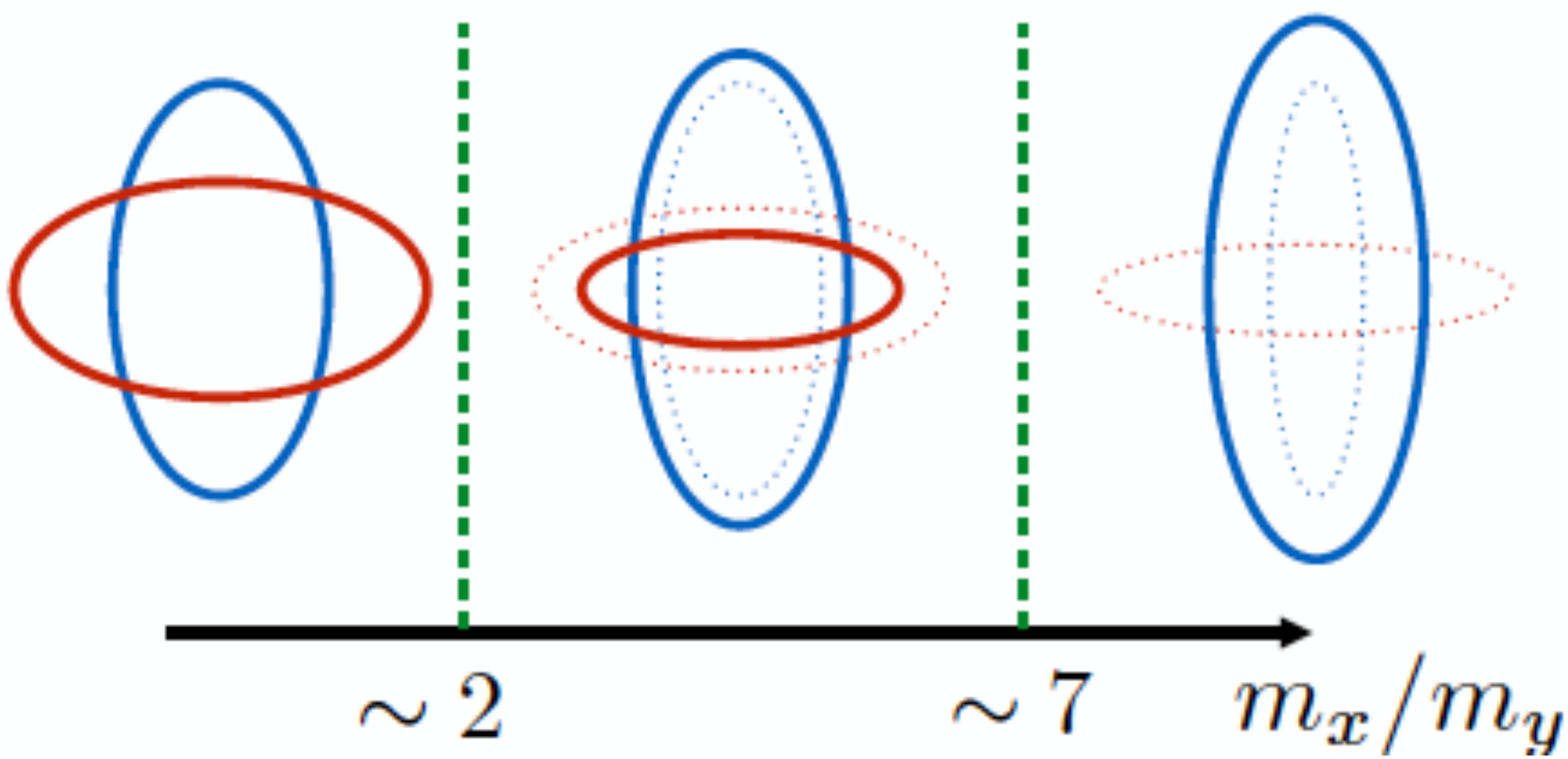
Exact diagonalization

DMRG phase diagram

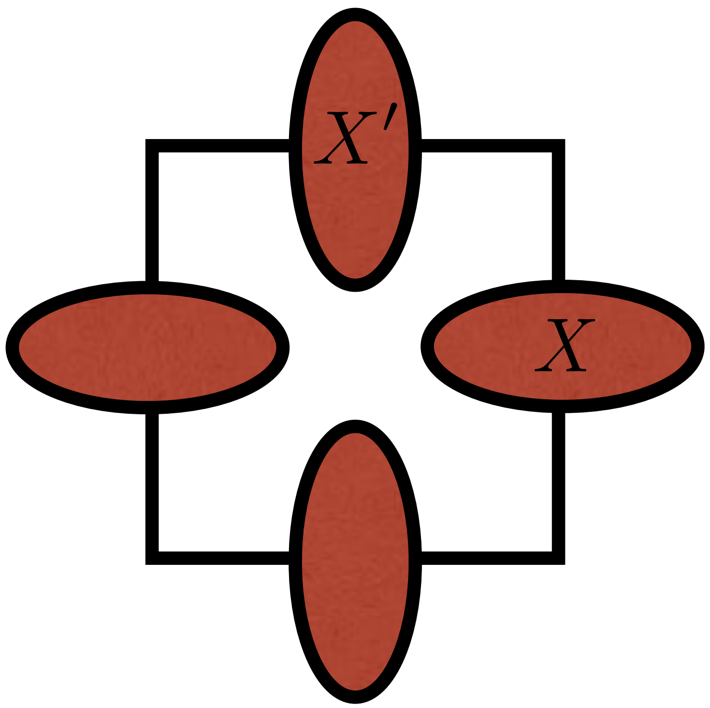
DMRG results:



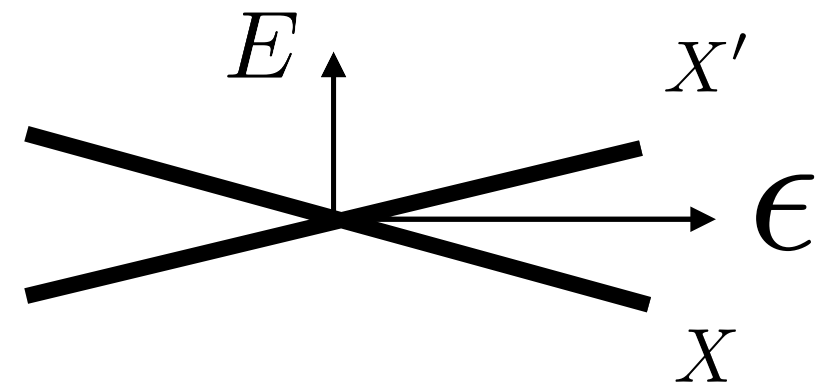
Valley Stoner magnet:



Connections with Aluminum Arsenide

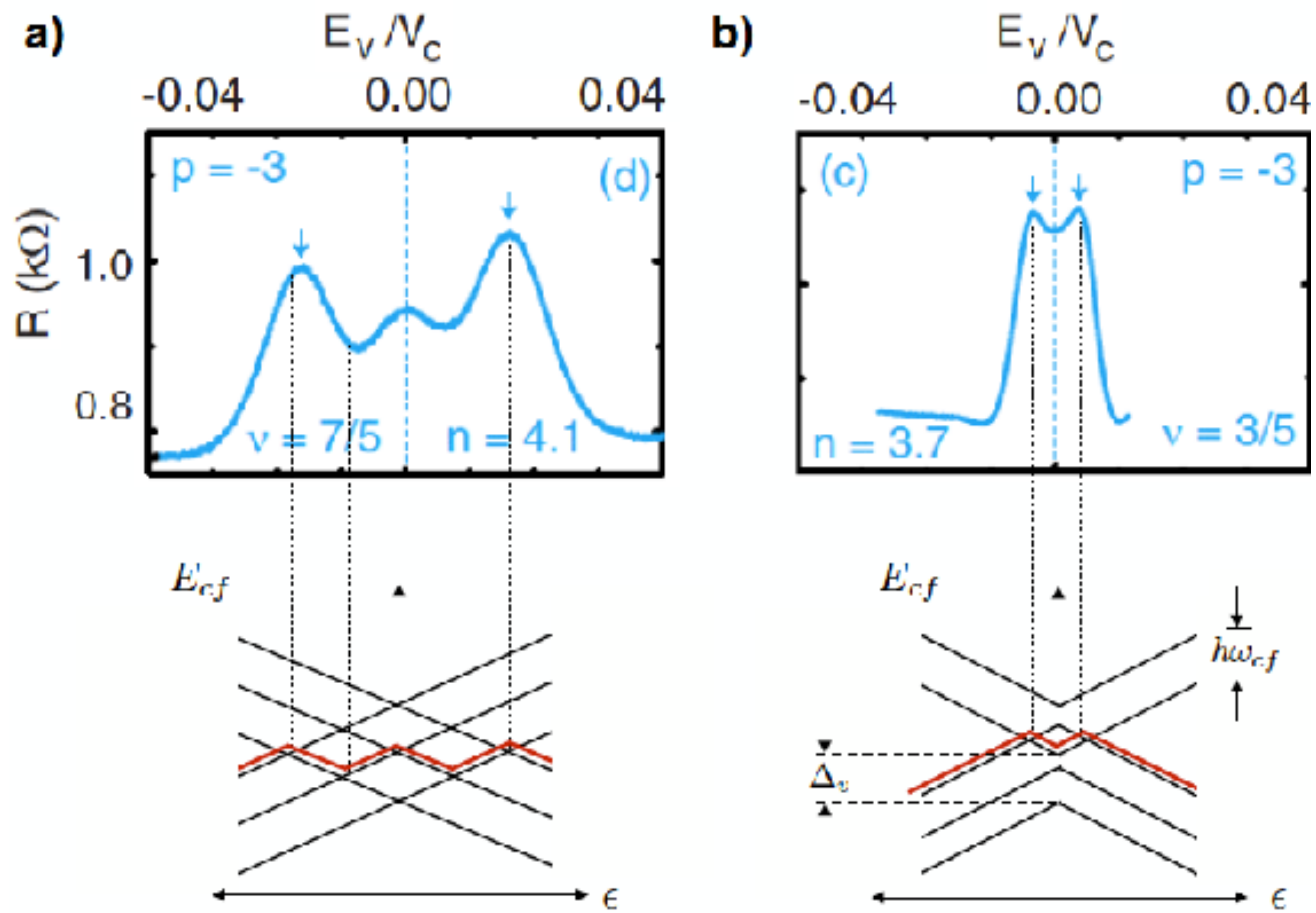


$$N = 0LL$$



$$\nu = 1/2$$

Nearby FQH states appear to have spontaneous valley polarization:



Padmanabhan, Gokmen, and Shayegan PRB (2010)