Numerical investigation of gapped edge states in fractional quantum Hall-superconductor heterostructures

Cécile Repellin

Rencontres du Vietnam

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Perspectives in topological phases

Quy Nhon, July 2018

Massachusetts Institute of Technology

MARIE CURIE ACTIONS

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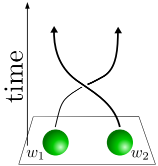
CR, A. Cook, T. Neupert, N.Regnault
NPJ Quantum Materials 2018

Motivation: non-abelian anyons

Exchange statistics of two particles

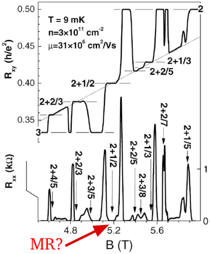


- $\Psi(w_2, w_1) = \pm\Psi(w_1, w_2)$ $\rightarrow$ boson, fermion
- $\Psi(w_2, w_1) = e^{i\theta}\Psi(w_1, w_2)$ $\rightarrow$ abelian anyon
- $\Psi_a(w_1, w_2), \Psi_b(w_1, w_2)$ ... degenerate manifold
  Braiding = rotation: $\Psi_i(w_2, w_1) = \sum_j U_{ij}\Psi_j(w_1, w_2)$
  $\rightarrow$ **non-abelian anyon**

Non-abelian modes in condensed matter systems?

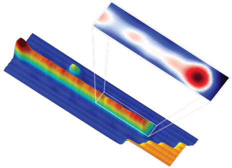
Topological order

emergent quasiparticles
free excitations



Localized topological mode

specifically engineered
pinned excitations



S. Nadj-Perge et. al. Science 2014

- Moore-Read (5/2) $\rightarrow$ Majorana
- Read-Rezayi (12/5) $\rightarrow$ $\mathbb{Z}_3$ parafermion

- Kitaev chain $\rightarrow$ Majorana
- abelian FQH / superconductor heterostructure??

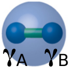
Outline

Boundary-induced topological degeneracies: the Kitaev chain

Kitaev chain

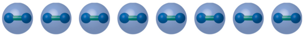
$$H = -(\sum_j \mu c_j^\dagger c_j + \frac{\Delta}{2} c_j^\dagger c_{j+1} + \frac{\Delta}{2} c_j c_{j+1} + h.c.)$$
$$= -\frac{i}{2} \sum_j (-\mu \gamma_{A,j} \gamma_{B,j} + \Delta \gamma_{B,j} \gamma_{A,j+1})$$

$$c_j = (i\gamma_{A,j} + \gamma_{B,j})/2$$

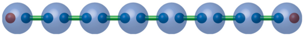


Majorana modes

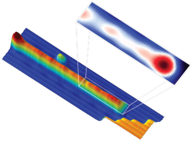
$$\gamma^2 = 1, \gamma = \gamma^\dagger, \gamma_j \gamma_i = -\gamma_i \gamma_j$$



- $\mu \gg \Delta$: no zero energy state



- $\mu \ll \Delta$
  2 zero-energy states localized at the edges



Experimental realizations:

- semiconductor nanowire (L. Kouwenhoven)
- magnetic atoms on supercond. (A. Yazdani)

Parafermions from FQH heterostructures

Going beyond Majoranas

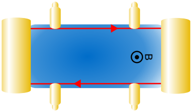
How to create richer topological order?

$\mathbb{Z}_k$ **parafermions**

- $\gamma^k = 1, \gamma^{k-1} = \gamma^\dagger$
- $k = 2$: Majorana

Physical system? Theoretical predictions:

- 1D bosonic/fermionic system: no parafermion
- 1D system of *fractionalized quasiparticles* coupled to superconductor
  → **edge of fractional quantum Hall state**



1D system at the edge of a 2D topological system → Not purely 1D!

Beyond effective theories

Theoretical predictions of emergence of parafermions are based on bosonized description of the edge (Luttinger liquid)

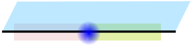
- Quantitative predictions in microscopic model?
  Length/energy scales?

Numerics: bridge between theory and experiment

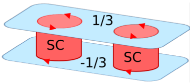
- Parafermion localized at domain wall of different edge phases

- 2D models with most symmetries broken
- Naive numerical implementation of proposed 2D models would be extremely limited in size

Do we need *point-like* excitations?



Non-abelian degeneracies without localized zero modes



Barkeshli PRL 2016

Bilayer FQH system with time-reversal symmetry coupled to supercond.

gapped mode delocalized along the edge

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Charge $2e/3$ Superconductivity and Topological Degeneracies without Localized Zero Modes in Bilayer Fractional Quantum Hall States

Maissam Barkeshli
Station Q, Microsoft Research, Santa Barbara, California 93106-6105, USA
Department of Physics, Condensed Matter Theory Center, University of Maryland, College Park, Maryland 20742, USA and Joint Quantum Institute, University of Maryland, College Park, Maryland 20742, USA
(Received 26 April 2016; published 24 August 2016)

Is this realistic?

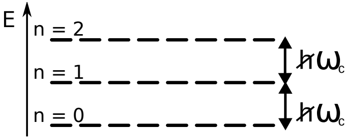
- B / $-B$ bilayer: graphene, $1/3 + 2/3$
- SC + large B?: *Supercurrent in the QH regime, Amet et. al. Nature 2016*
  But FQH + SC interface remains a hard experimental challenge

Go beyond Luttinger liquid: 2D microscopic model

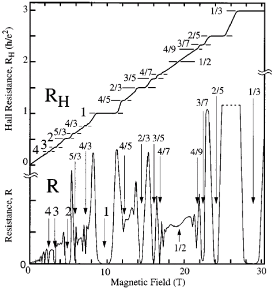
The fractional quantum Hall effect

Electrons in a magnetic field in 2D

Single-particle picture: Landau levels (LL)



filling fraction $\nu = N/N_\phi$



Many-body system

- $\nu = n$ integer: n filled LL → Integer quantum Hall effect
- $\nu < 1$: fractionally filled LL → Fractional quantum Hall effect

$t_{disorder} \ll V_{int} \ll \hbar\omega_c$
(verified for large B)

We restrict the analysis to the lowest LL: effective Hamiltonian $P_{LLL}\hat{V}_{int}P_{LLL}$

Two main approaches:

- Numerical: exact diagonalization, DMRG
- Ansatz wave function

The Laughlin wave function

One-body wavefunction in the lowest Landau level (symmetric gauge):

$$\varphi_{\mathbf{m}}(\mathbf{z}) \propto \mathbf{z}^{\mathbf{m}} \mathbf{e}^{-|\mathbf{z}|^2/4\mathbf{l_B}^2}, \quad z = x + iy$$

N particles in the LLL at positions $z_1, ..., z_N$

$\nu = 1/3$ Laughlin state:

$$\boldsymbol{\Psi}_{\mathbf{Lgh}}(\mathbf{z_1}, ..., \mathbf{z_N}) = \prod_{\mathbf{i<j}} (\mathbf{z_i} - \mathbf{z_j})^{\mathbf{3}} \mathbf{e}^{-\sum_{\mathbf{i}} |\mathbf{z_i}|^2/4\mathbf{l_B^2}}$$

- very accurate description of realistic ground state (first observed fraction)
- unique densest ground state of model interaction
- convenient mathematical properties

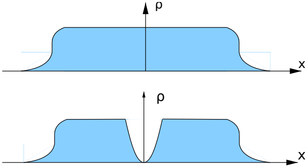
degeneracy on higher genus surface: 3-fold degeneracy on the torus

Quasihole excitations of the Laughlin wave function

Laughlin state plus one flux quantum at w_1:

$$\boldsymbol{\Psi_{qh}(z_1, ..., z_N) = \prod_i (z_i - w_1) \Psi_{Lgh}(z_1, ..., z_N)}$$

- Locally, creates quasihole of charge $+e/3$
- quasielectron: charge $-e/3$



Two quasiholes at w_1, w_2:

$$\Psi^{2qh} = \mathcal{N} \, . \, (w_1 - w_2)^{-1/3} \, . \, \left(\prod_j (z_j - w_1)(z_j - w_2) \right) \, . \, \Psi_{Lgh}(z_1, ..., z_N)$$

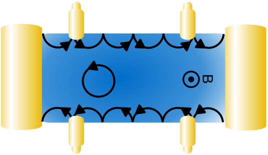
$\rightarrow$ **fractional exchange statistics**

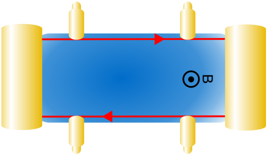
Fractionalization: characteristics of intrinsic topological order

Edge states of a fractional quantum Hall (Laughlin) system

Edge excitations of a Laughlin $\nu = 1/3$ state

- **Semi-classical picture**



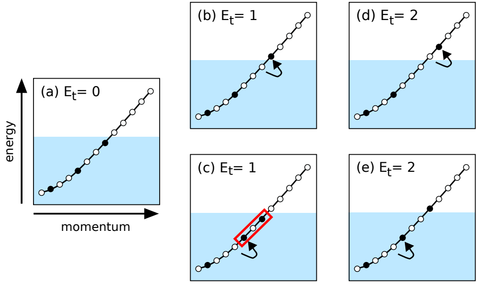


- **Landau level with confinement potential**

  quasihole excitations

  $\leftrightarrow$

  edge excitations

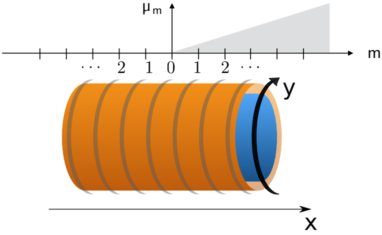


Edge excitations of a Laughlin $\nu = 1/3$ state

Energy spectrum
$\rightarrow$ chiral edge mode with linear dispersion relation

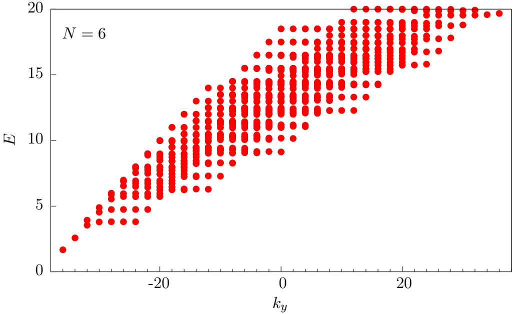
Confinement on one side only

Cylinder with linear confining potential



- m = single-particle momentum along y axis
- $k_y = \sum_m m \; n_m$

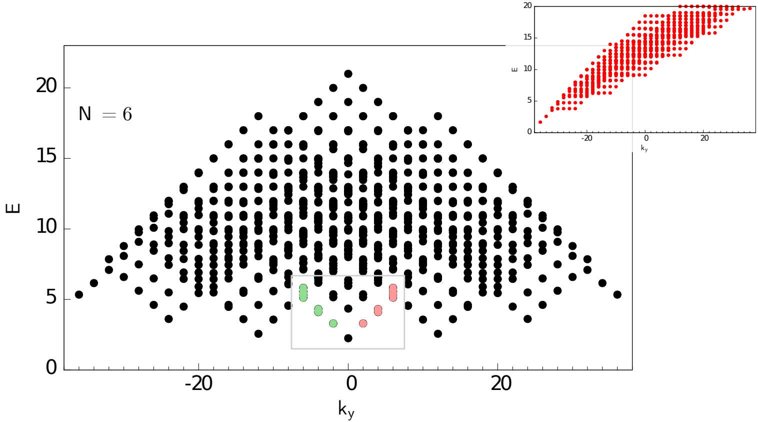
one chiral mode



Edge excitations of a Laughlin $\nu = 1/3$ state

Symmetric confinement potential

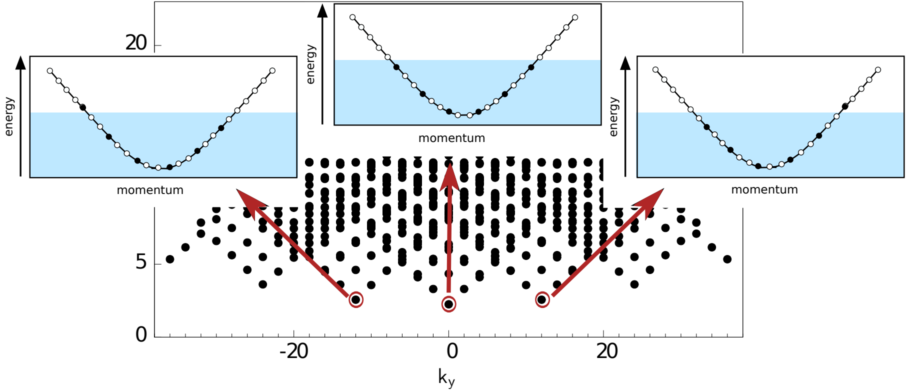
- two (opposite) chiral modes



Edge excitations of a Laughlin $\nu = 1/3$ state

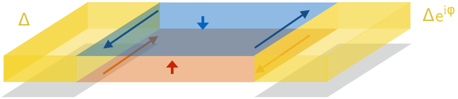
Symmetric confinement potential

- "replicas": macroscopic momentum transfer, small energy difference



Non-abelian modes at the boundary of a fractional quantum Hall bilayer

The physical model



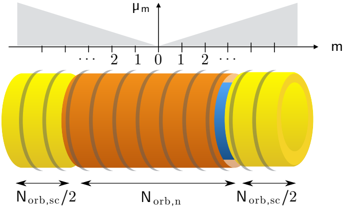
- $1/3 \otimes -1/3$ FQH state
- s-wave superconductor $(\Delta c_\downarrow c_\uparrow + h.c.)$ couples two layers at the edge

At the edge

- counterpropagating Luttinger liquids with bosonic fields $\phi_\uparrow$, $\phi_\downarrow$
- electron operators: $c_{\downarrow,\uparrow} \sim e^{i3:\phi_{\downarrow,\uparrow}:}$
- Andreev backscattering term $\mathcal{L}_t = -\Delta \cos(3(\phi_\uparrow + \phi_\downarrow))$
- Condensation of Cooper pairs of Laughlin quasiparticles $e^{i:\phi_\uparrow + \phi_\downarrow:}$
- **Gapped ground state with 3-fold degeneracy**

Quantitative analysis of the competing energy/length scales in a microscopic model?

Our microscopic setup



$$
\begin{aligned}
H \quad = \quad & H_{\text{int},\uparrow} + H_{\text{int},\downarrow} && \rightarrow 1/3_{\uparrow} \otimes -1/3_{\downarrow} \\
& + \sum_{m,\sigma} \mu_m c^{\dagger}_{m,\sigma} c_{m,\sigma} && \rightarrow \text{confining potential} \\
& + \frac{1}{2C} \left(\hat{N} - \hat{N}_0 \right)^2 && \rightarrow \text{charging energy} \\
& + \Delta_0 \sum_m f_m c^{\dagger}_{m,\uparrow} c^{\dagger}_{m,\downarrow} \; + \; h.c && \rightarrow s\text{-wave pairing}
\end{aligned}
$$

How to reach sufficient sizes with numerical simulations?

(particle number / translation along cylinder not conserved)

Project onto the subspace of Laughlin quasiholes

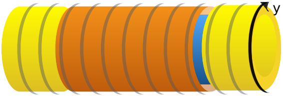
- Laughlin quasiholes wf are Jack polynomials
- Well known expression in occupation basis
- Hamiltonian is efficiently written in quasihole subspace
- **Exact diagonalization of the Hamiltonian in the quasihole subspace**

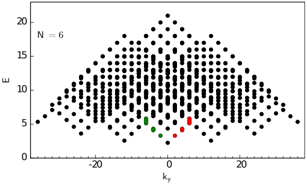
Bulk Laughlin gap sent to infinity

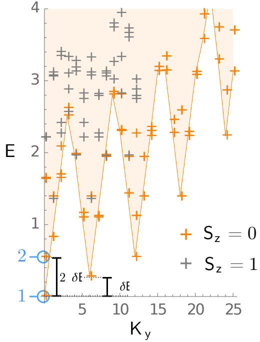
Bulk screening properties remain through the Laughlin correlation length

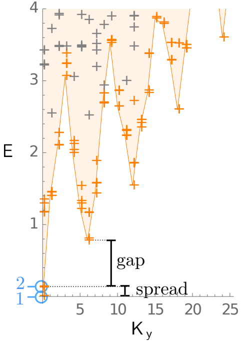
Energy results: gapping out the edges

$N_\phi = 21$
$L_y/l_B = 15$







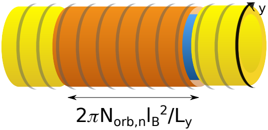


No superconductivity ($\Delta_0 = 0$)
Gapless with finite-size splitting

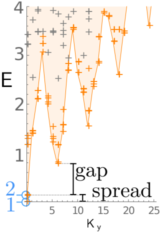
$\Delta_0 = 2$
$\sim$ **3-fold quasidegeneracy**

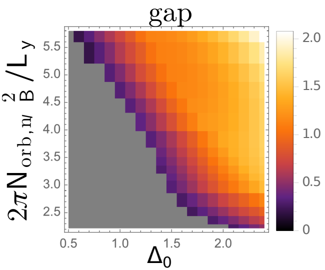
Energy results: gapping out the edges

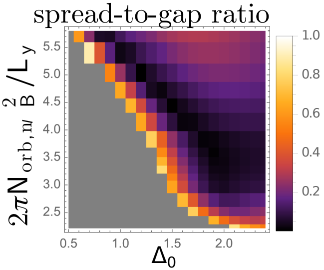
$N_\phi = 21$
$L_y/l_B = 15$



$\sim$ length of normal region

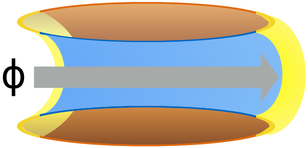




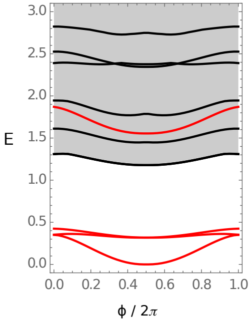


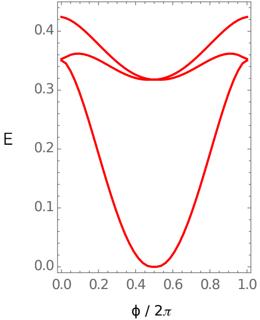
How do we probe the topological character of the degeneracy?

Spectral flow under spin-flux insertion



Charge pumping experiment (Laughlin's Gedanken experiment)

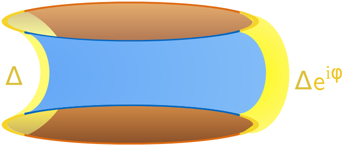




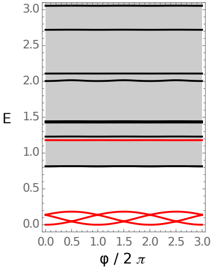
For each inserted flux quantum
a charge $2e/3$ is transfered from one edge to the other

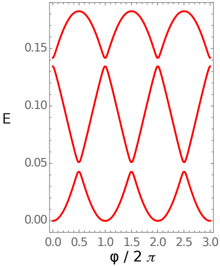
$2e/3$ Josephson effect

SC creates condensate of $2e/3$ quasiparticles at the edges



- topologically trivial: $\varphi = 0 \to 2\pi$ tunnels charge **2e**
- $(1/3)$ $(-1/3)$ bilayer: $\varphi = 0 \to 2\pi$ transfers charge **2e/3**



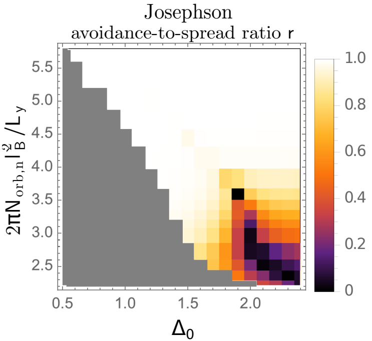


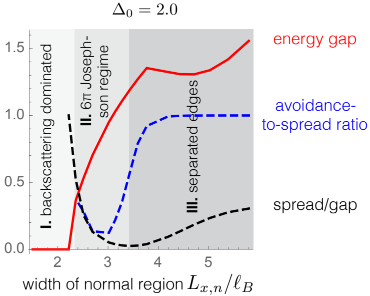
- 2π: permutes the topological sectors
- 6π: goes back to the original situation

Fractional Josephson effect

Josephson effect quality factor

$$r := \frac{\text{largest avoided crossing}}{\text{spread of the ground state manifold}}$$





Hierarchy of correlation lengths:

- $2\pi N_{orb,n}\ell_B^2/L_y \gg \ell_{Laughlin} \simeq 1.4\ell_B$: tunneling through the gapped bulk exponentially suppressed
- $2\pi N_{orb,n}\ell_B^2/L_y \sim \ell_{edge}$: edge gap closes

Optimal region where all hypotheses are verified

Conclusion

- Towards engineering of non-abelian excitations in condensed matter systems
- Abelian FQH/superconductor boundary: first signatures in a microscopic model
- A convenient numerical setup
  - Setup can be adapted to matrix product states (larger sizes)
  - Edges of other FQH states (e.g. Moore-Read)?
- Direct evidence of non-abelian statistics?