Topological Features on a Triangular Lattice

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in collaboration with

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QUY NHƠN, VIỆT NAM
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Outline of Talk

- ✓ Motivation
- ✓ Band structure of a triangular lattice
- ✓ Anomalous Hall effect
- ✓ Optical control
- ✓ Outlook

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Dirac-Weyl Materials

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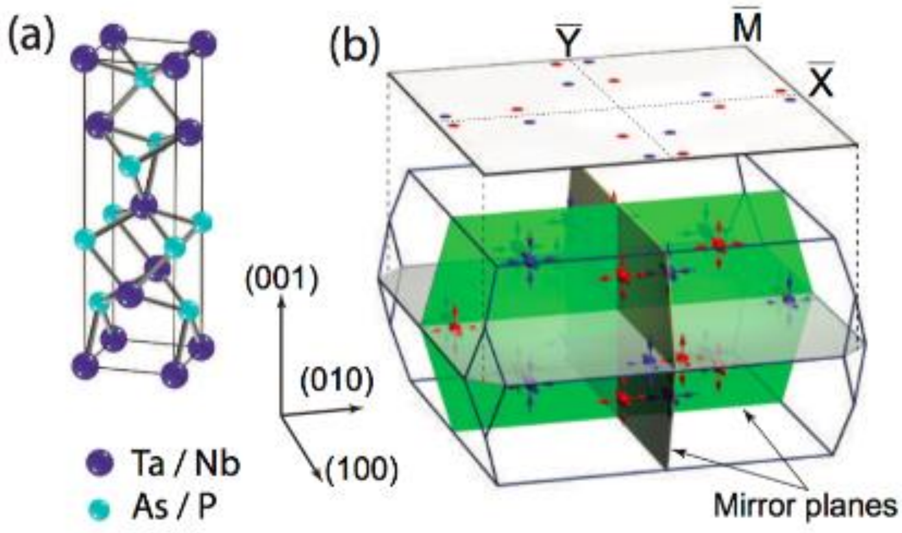
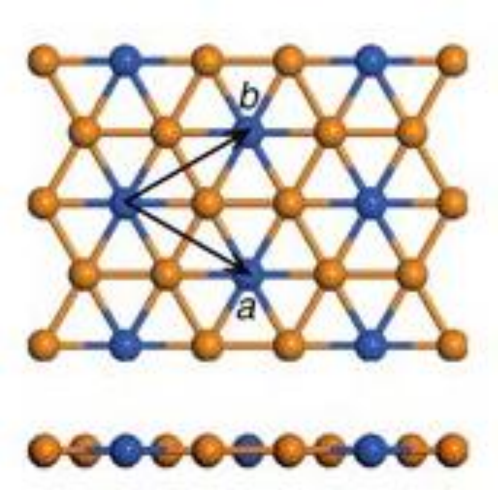


Crystal structure (space group I41md, No. 109)

Armitage, N. P., E. J. Mele, and Ashvin Vishwanath. "Weyl and Dirac semimetals in three-dimensional solids." *Reviews of Modern Physics* 90.1 (2018): 015001.

Experimental Motivation



- Cu
- Si

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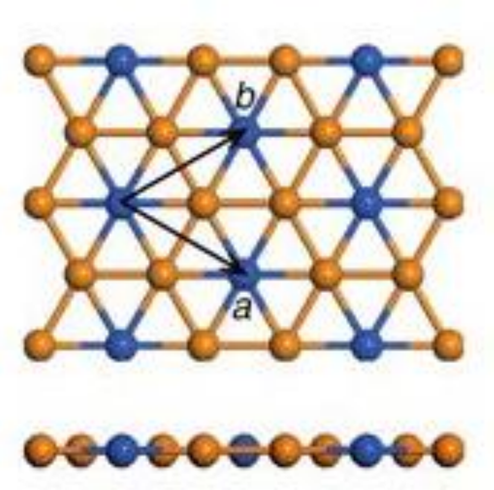

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In free-standing form, the lattice is invariant under *z*-mirror symmetry.

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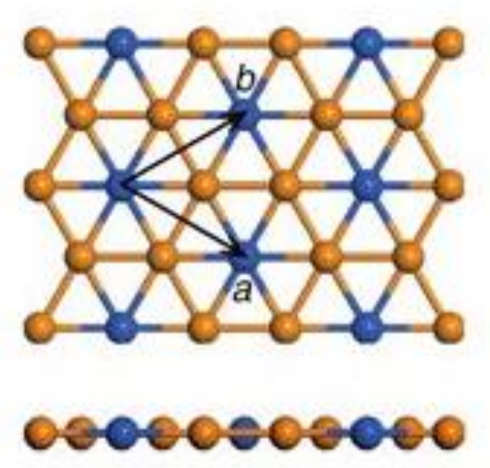

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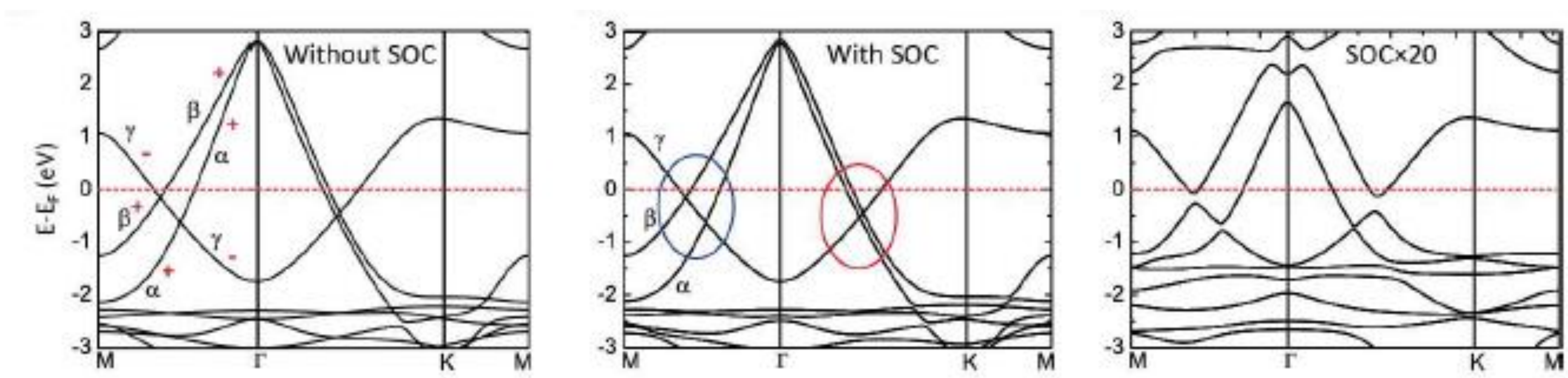

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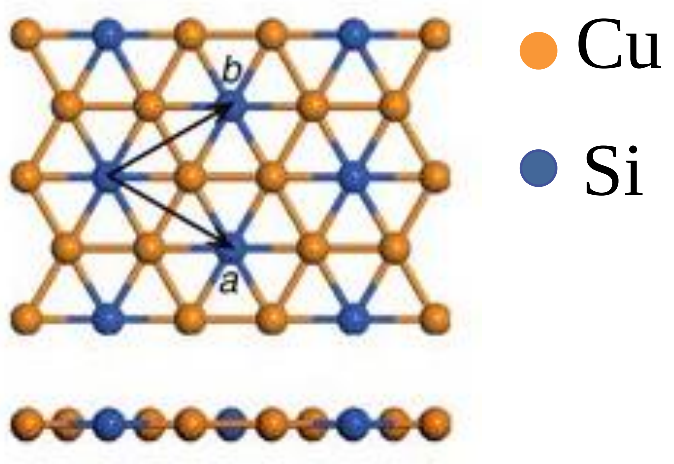
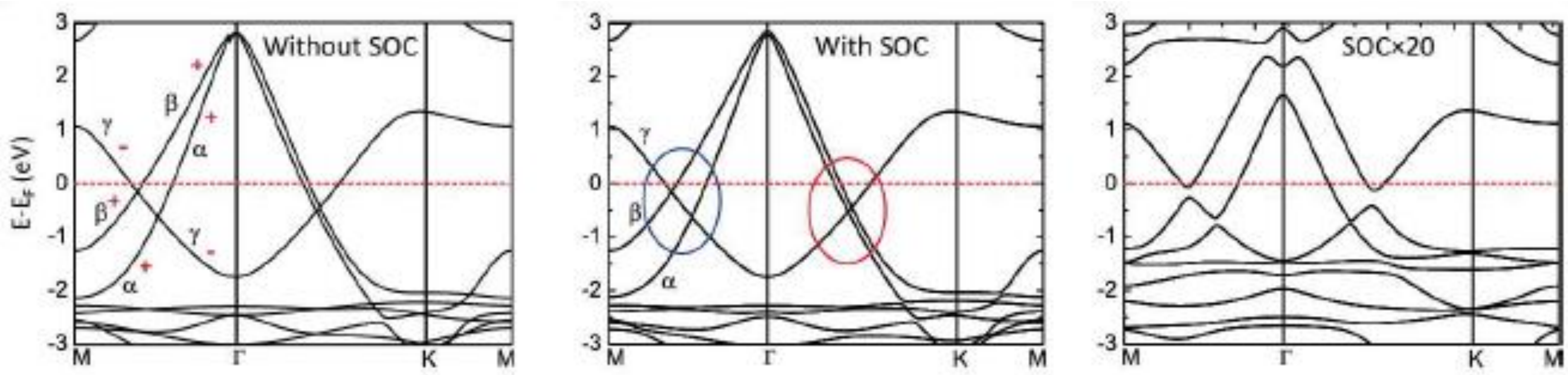


In free-standing form, the lattice is invariant under *z*-mirror symmetry.

Line nodes are protected by mirror symmetry in the absence of SOC.



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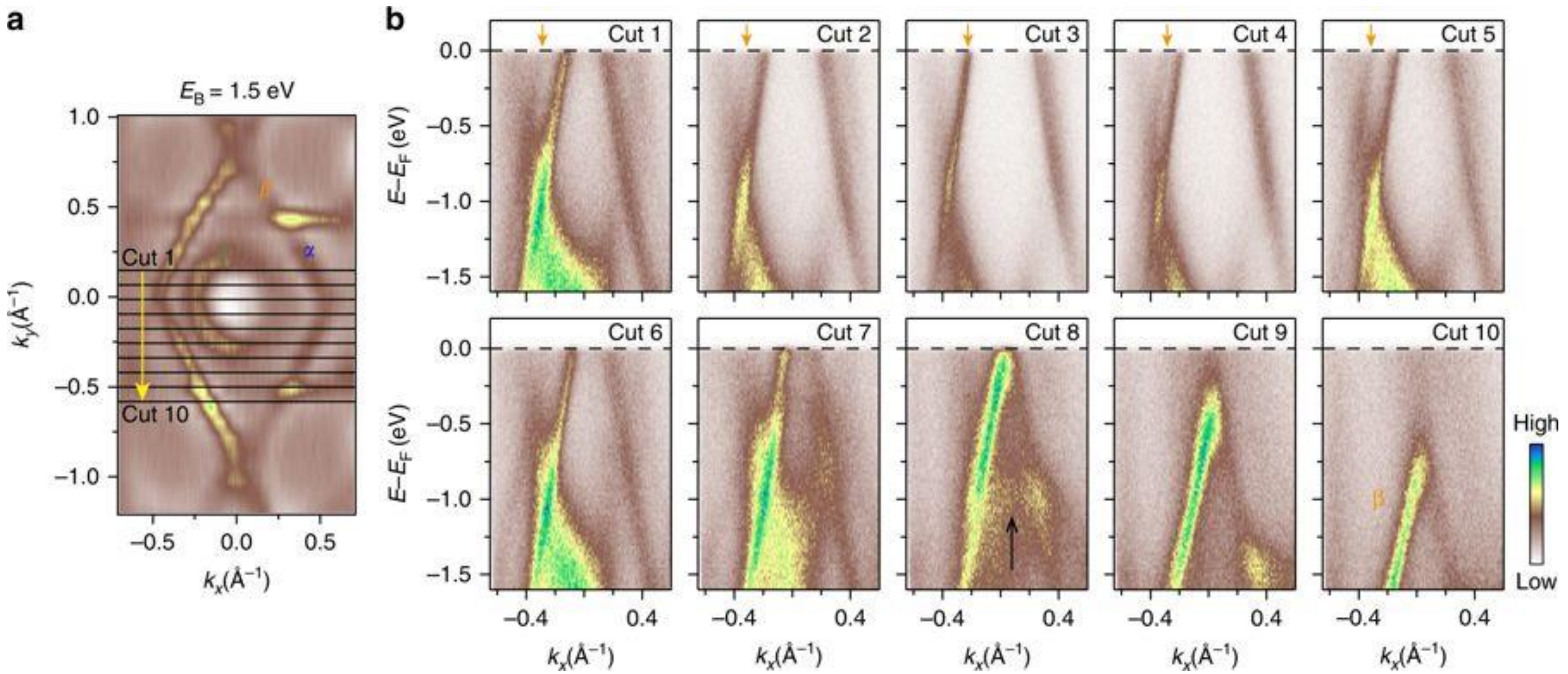
Experimental Motivation

Cu_2Si on Cu(111) ARPES Measurements



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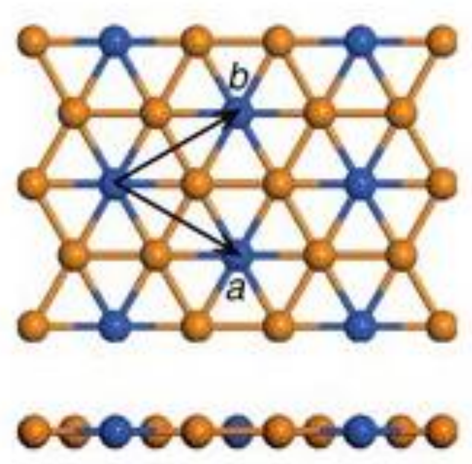
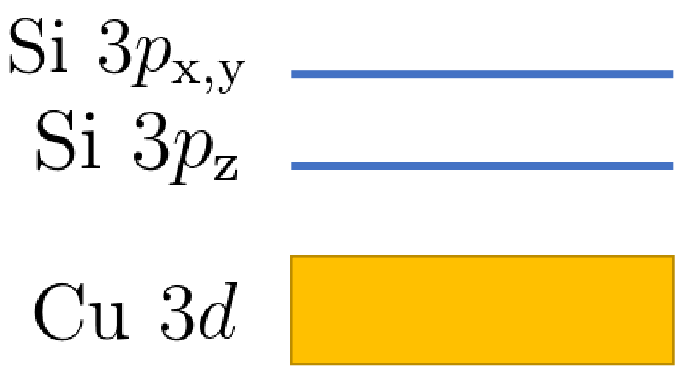
Minimal Band Theory





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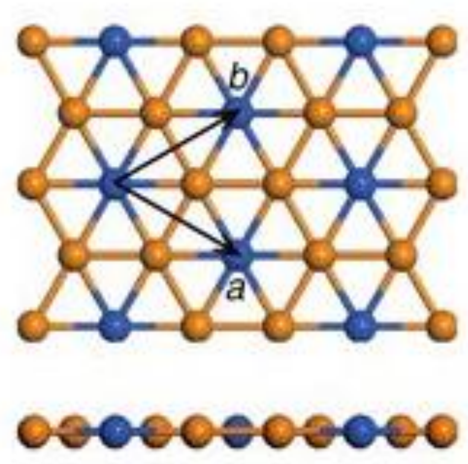

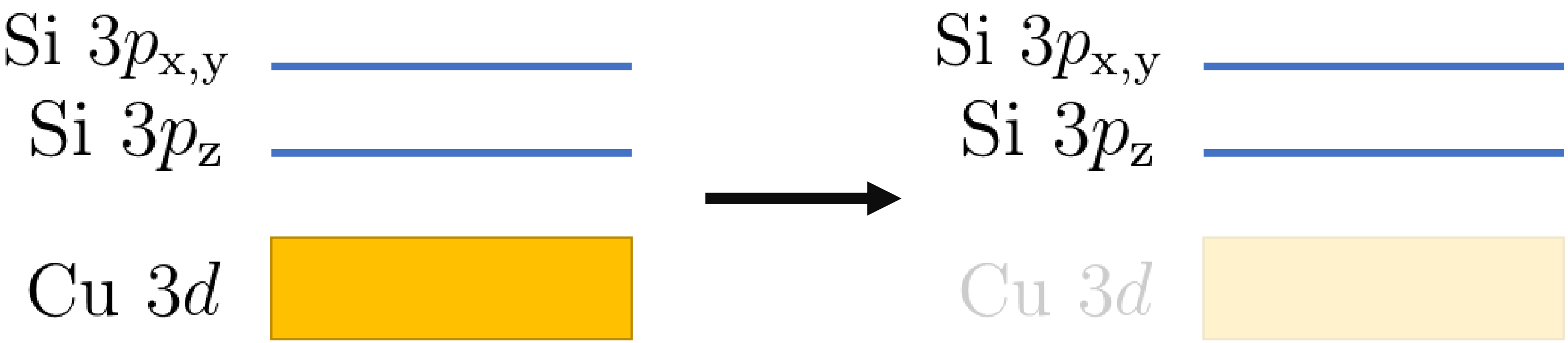

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Statement of Purpose

To explore the topological properties of a triangular lattice with a vector degree of freedom on each site

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- ✓ Anomalous Hall effect
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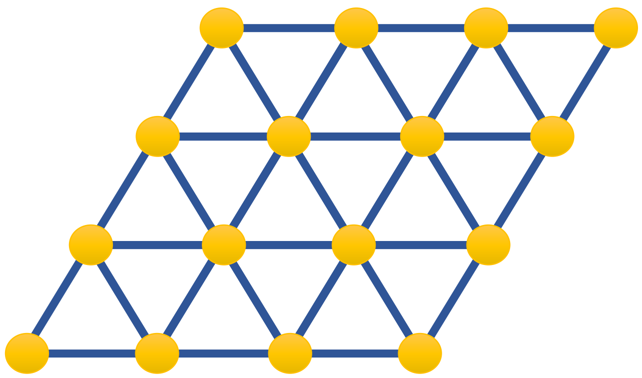
- ✓ Outlook

Hopping Matrix Elements



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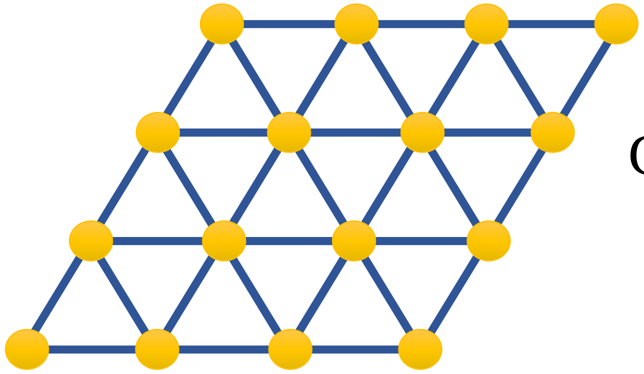
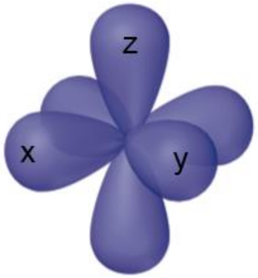


Cartesian Representation:



$$T(\phi) = \begin{pmatrix} t_{xx} & t_{xy} & 0 \\ t_{yx} & t_{yy} & 0 \\ 0 & 0 & t_{zz} \end{pmatrix}$$

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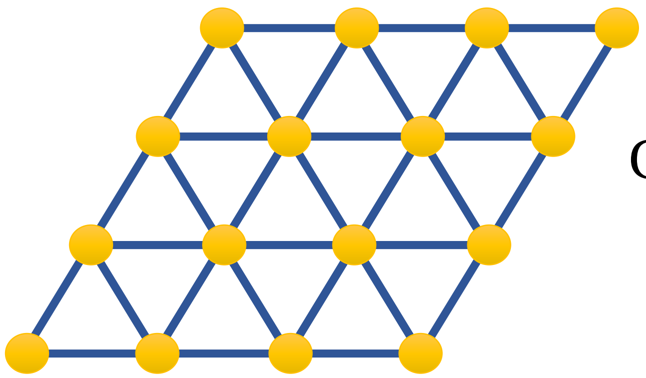

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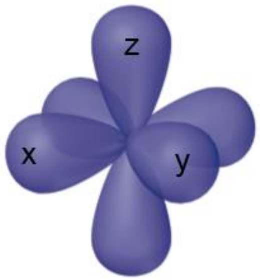

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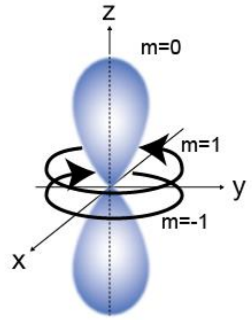

$$\Gamma(\phi) = \begin{pmatrix} \gamma_{-1,-1} & 0 & \gamma_{-1,1} \\ 0 & \gamma_{0,0} & 0 \\ \gamma_{1,-1} & 0 & \gamma_{1,1} \end{pmatrix}$$

k-space Hamiltonians

Cartesian Representation:

$$\mathcal{H}_{xyz}(\mathbf{k}) = h_0(\mathbf{k})\mathcal{I} + \Delta(\mathbf{k})L_z \cdot L_z + \mathbf{h}(\mathbf{k}) \cdot \mathbf{L}$$

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- **Line nodes** occur at intersections of $L = 0$ and orthogonal manifolds

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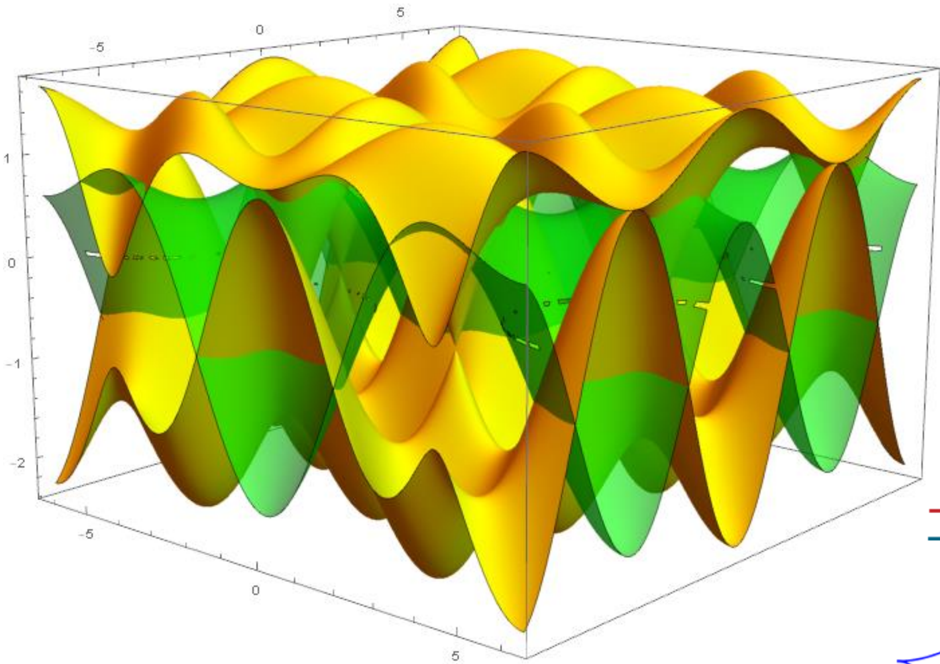
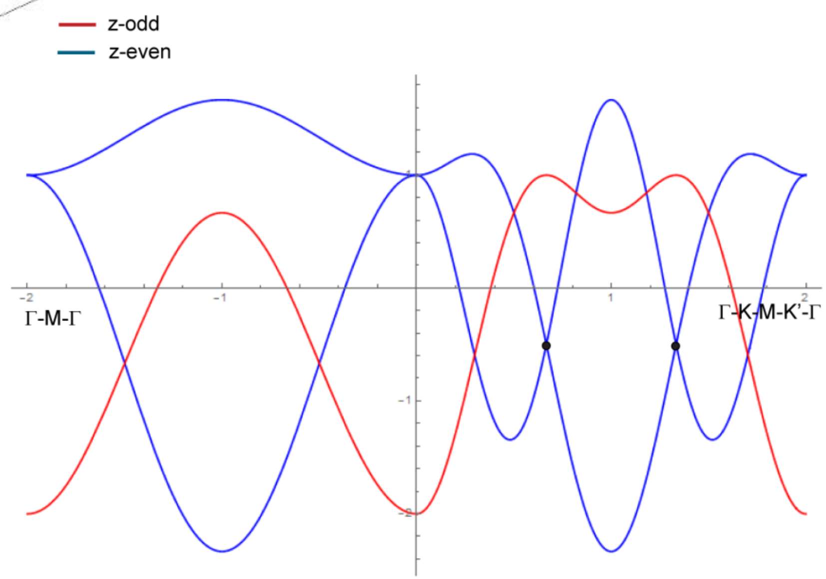
- **Line nodes** occur at intersections of L = 0 and orthogonal manifolds
- Other nodes occur at **exceptional points**

k-space Hamiltonians





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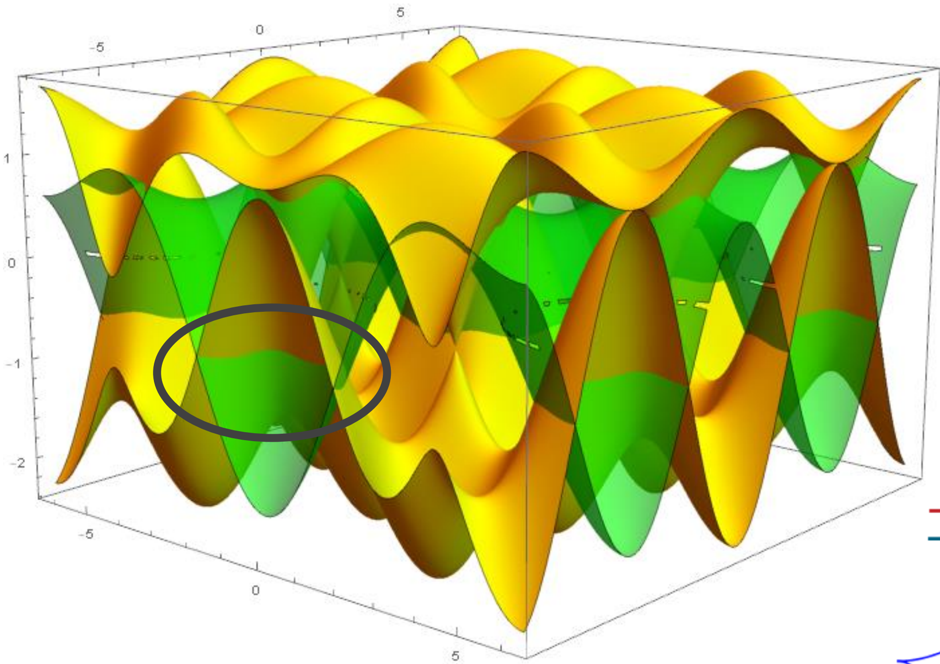

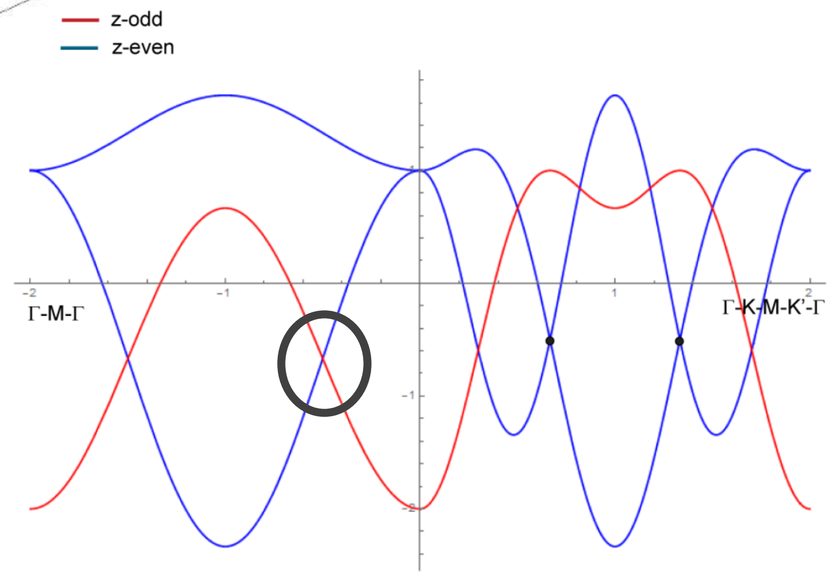

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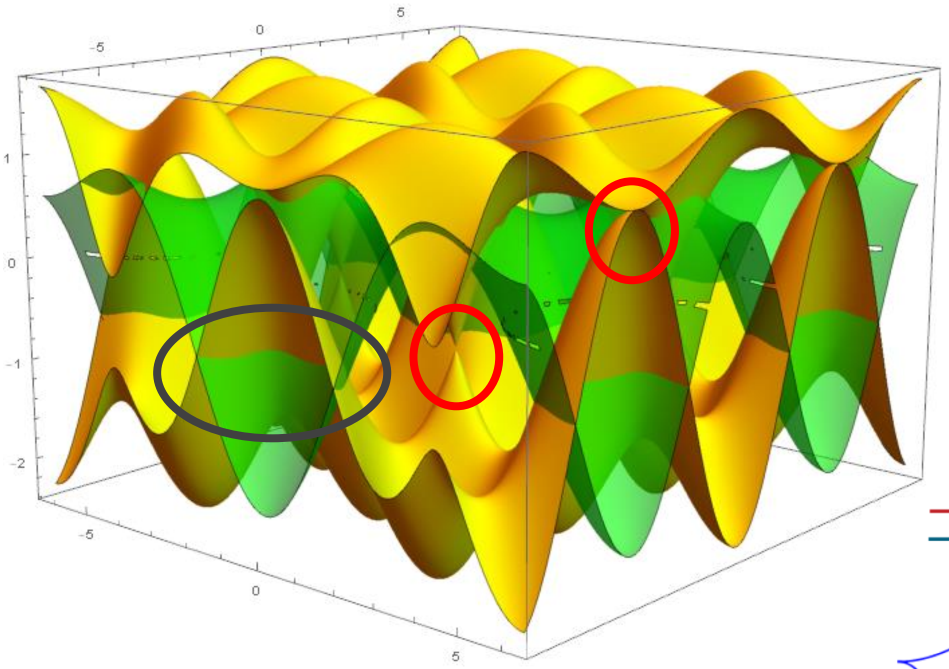

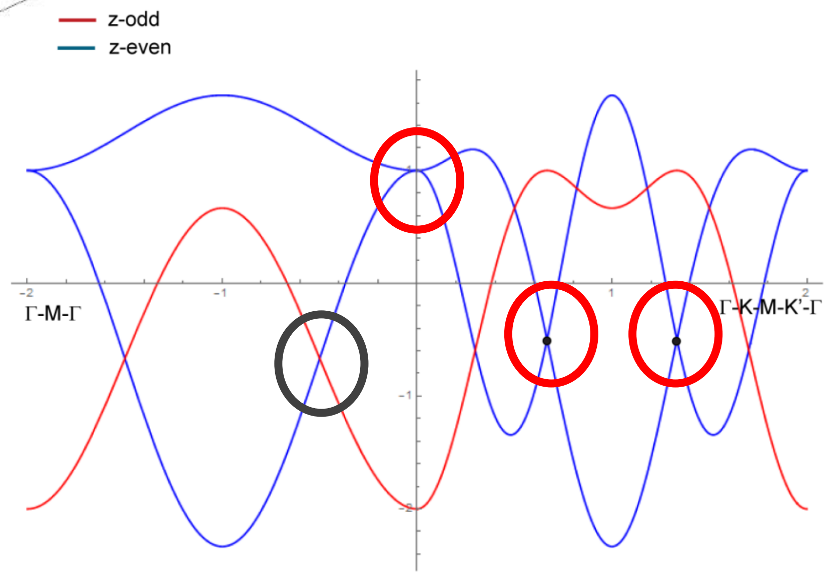

$|L|$=1 Manifold

Axial Representation:

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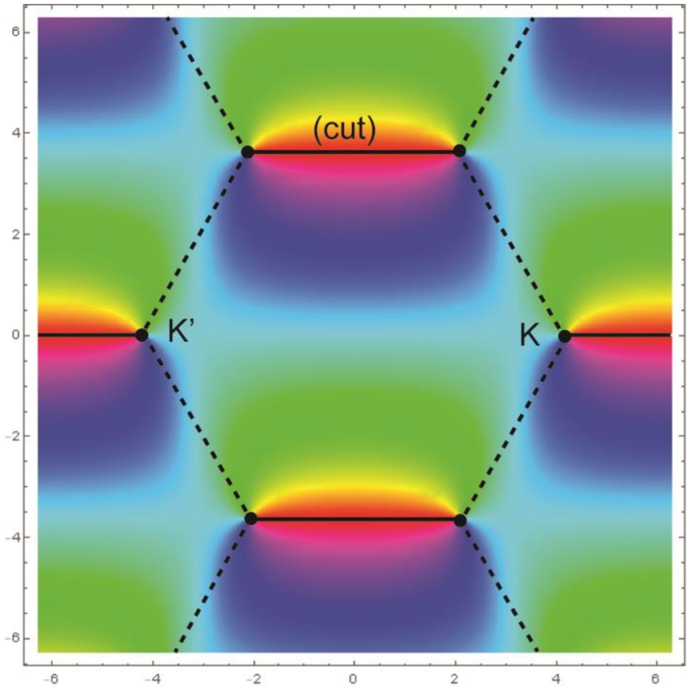
$$\mathcal{H}_{l_z=\pm 1}(q) = \begin{pmatrix} 0 & q_-^2 \\ q_+^2 & 0 \end{pmatrix}_\Gamma, \begin{pmatrix} 0 & q_+ \\ q_- & 0 \end{pmatrix}_{K,K'}$$

Comparison to Graphene

$$\arg(d(\mathbf{k}))$$

Graphene: J = ±1 nodes



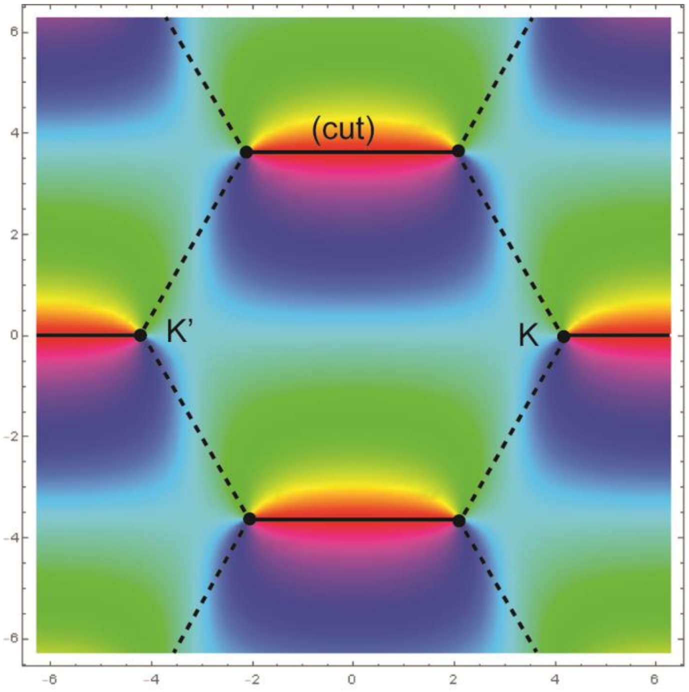
Partners at K and K'

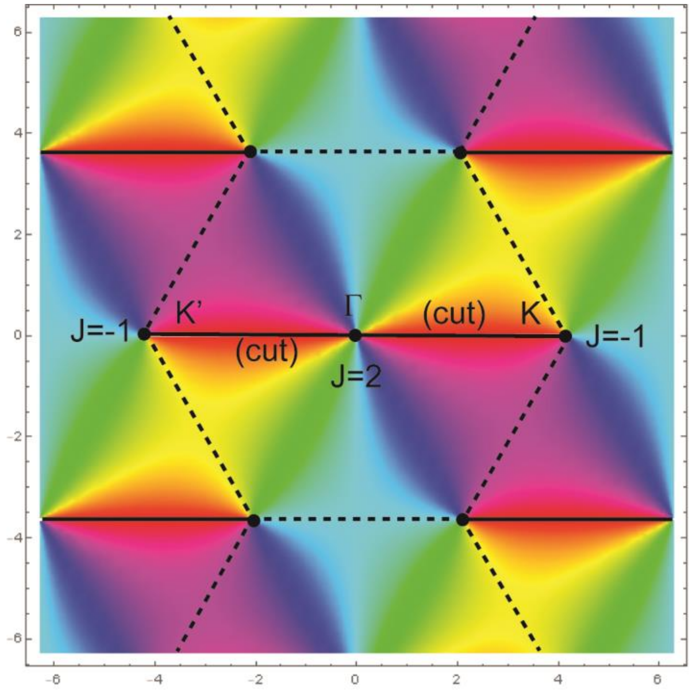
Comparison to Graphene

$$\arg(d(\mathbf{k}))$$

Graphene: J = ±1 nodes

T-lattice: J = -1,2 nodes





Partners at K and K'

Partners at K, K', and Γ

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Gapping Degeneracies

Recall that in order to observe anomalous charge Hall transport on the honeycomb lattice, one must use valley-antisymmetric mass terms:

$$\mathcal{H}_{\text{Haldane}} \propto \tau_z \sigma_z$$
$$\mathcal{H}_{\text{Kane-Mele}} \propto \tau_z \sigma_z s_z$$

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Or using non-equilibrium states via valley polarization.

However, in our model, anomalous charge transport can be directly accessed using site-localized, valley-symmetric perturbations.

Gapping Degeneracies

$$\sigma_z \text{ perturbation}$$

Breaks time-reversal symmetry, but preserves mirror symmetry

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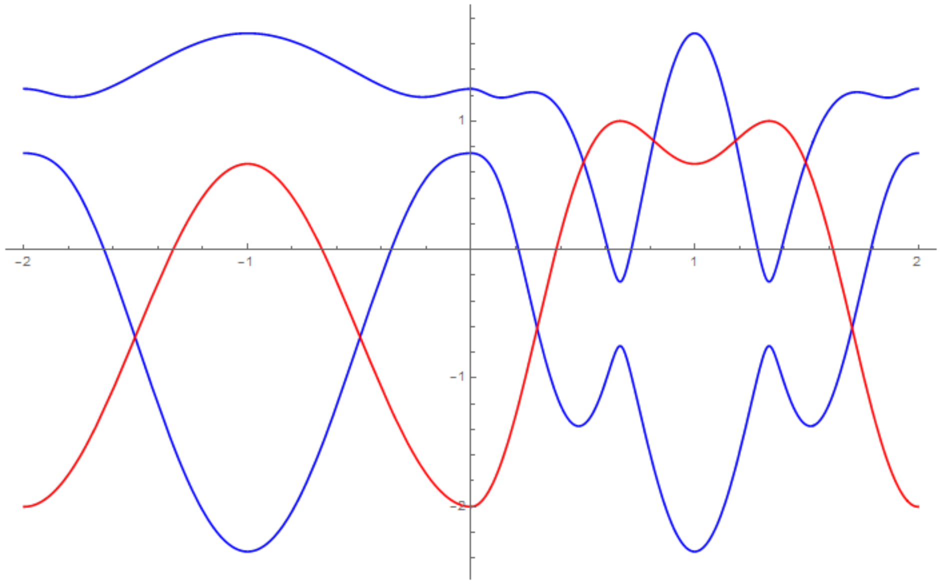
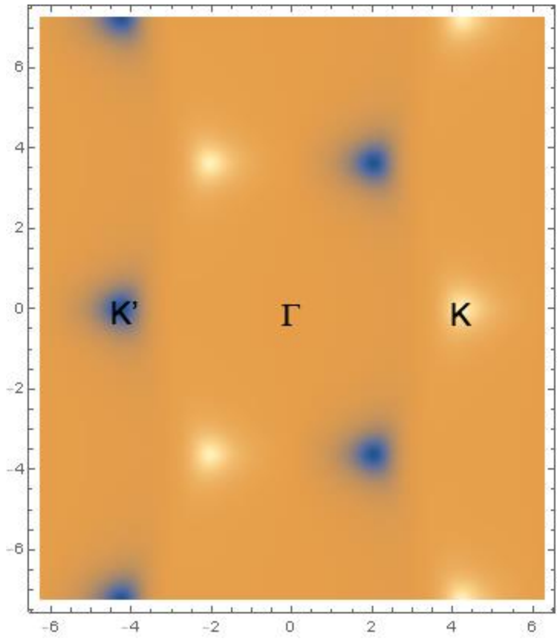


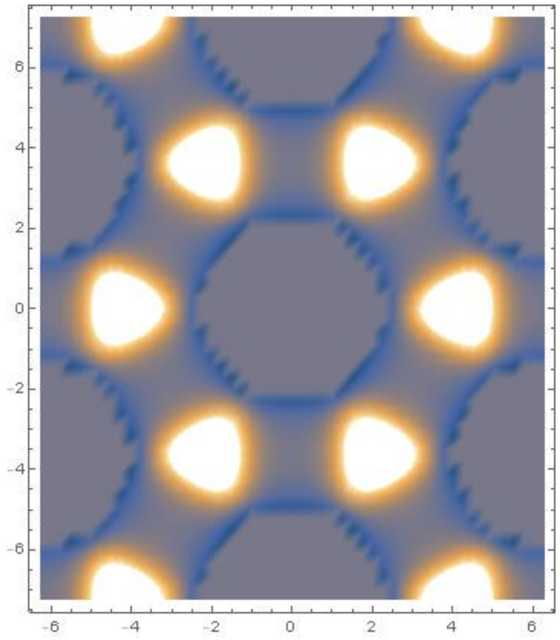
Anomalous Hall Effect

Berry Curvature

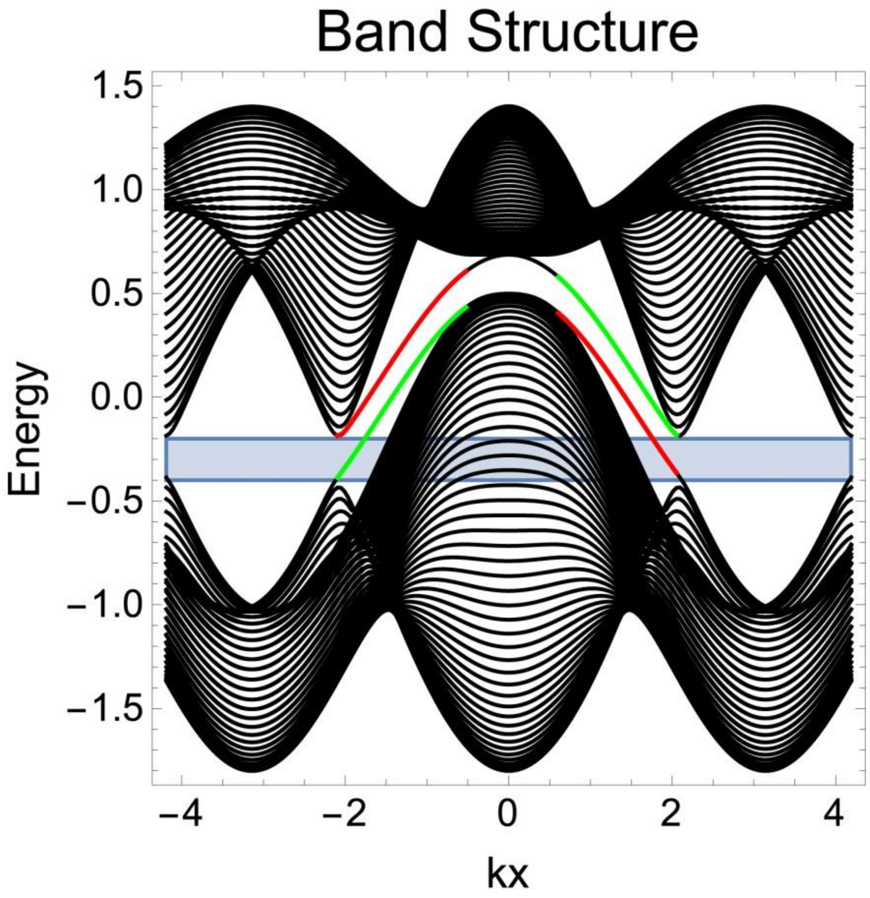
Graphene



Triangular lattice



Hall effect weakly screened



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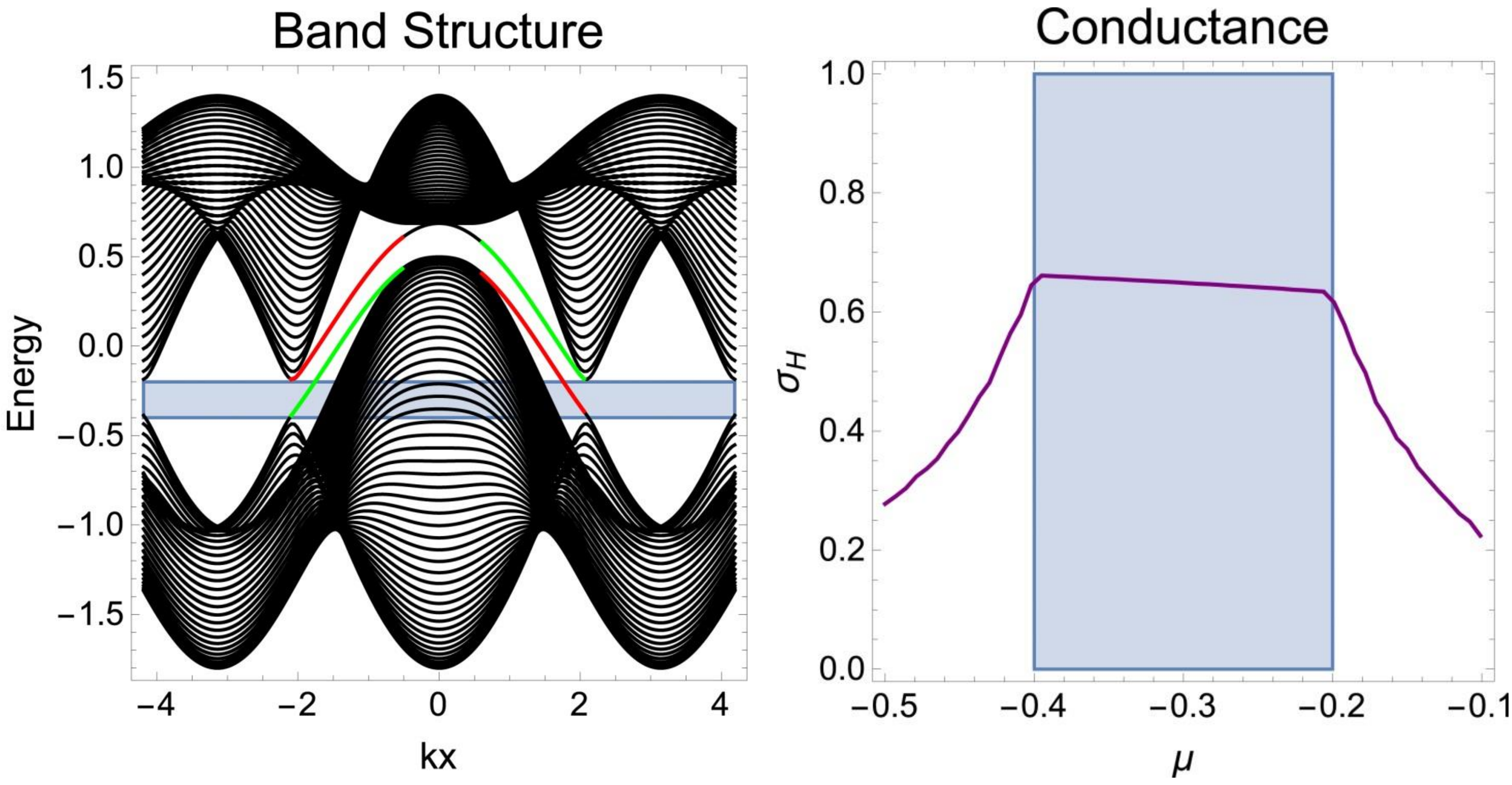
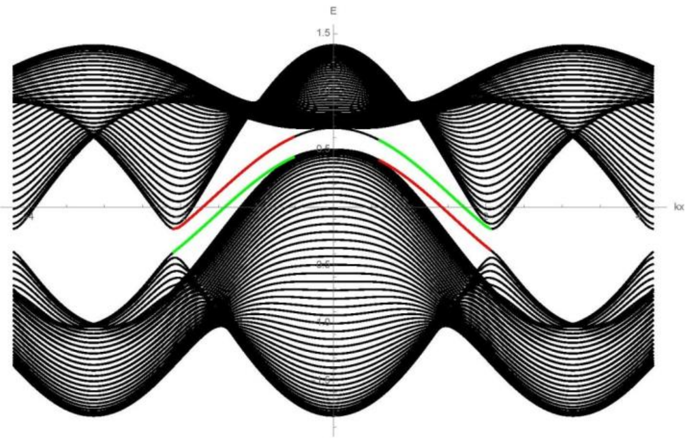
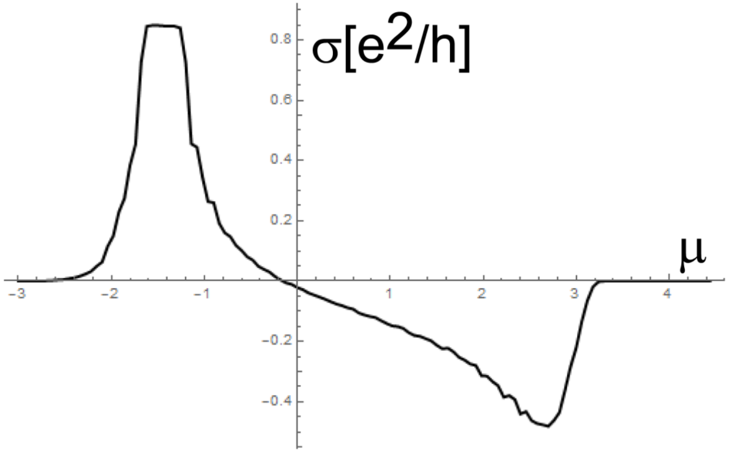


Anomalous Hall Effect

Weak coupling





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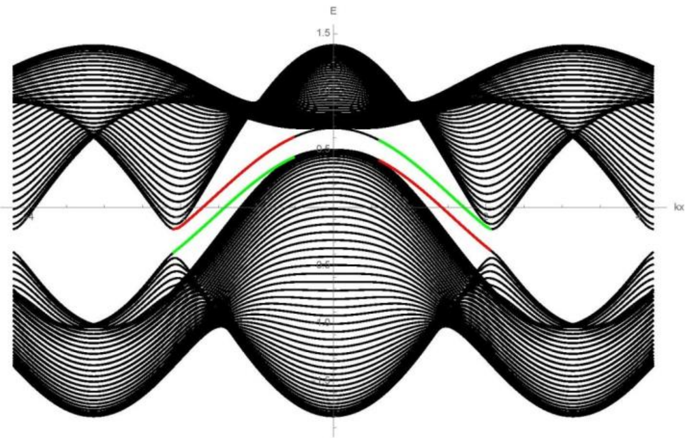
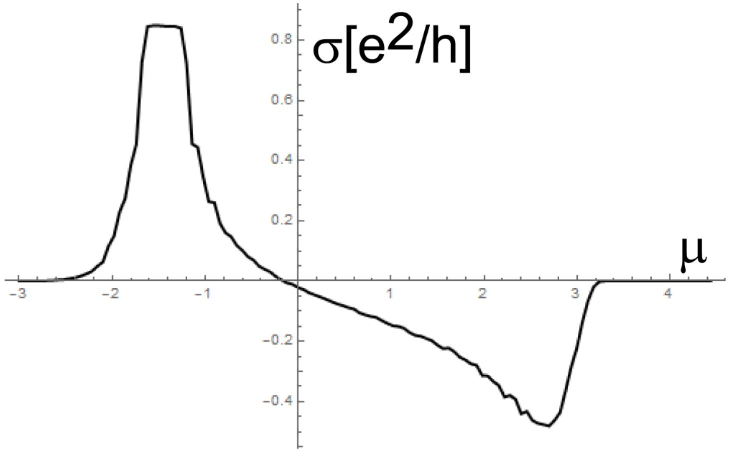
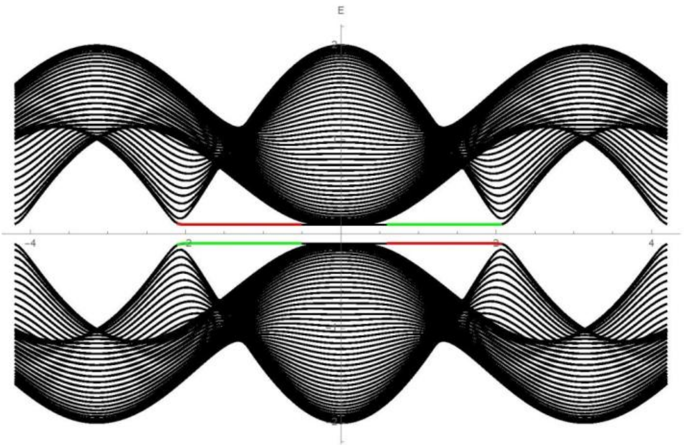
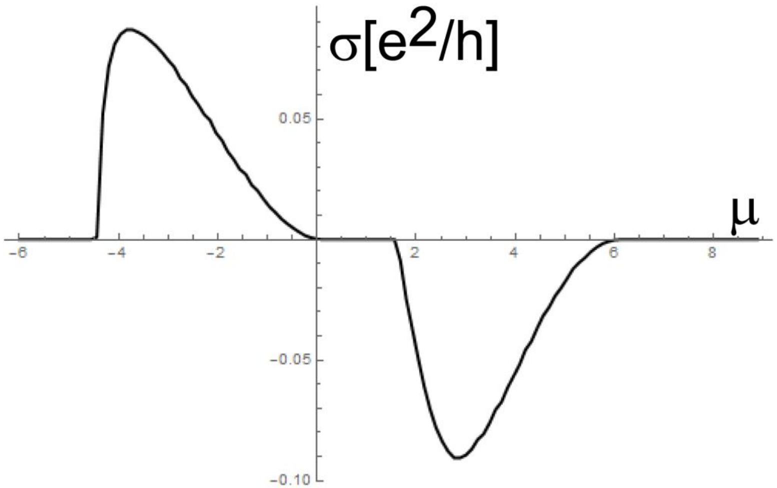




Strong coupling





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Exposure to Circularly Polarized Light

Dipole Approximation:

$$\mathcal{H}_{\text{int}}(t) = -e\mathbf{E} \cdot \mathbf{r}$$

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$$\mathcal{H}^{z}_{\text{int}}(t) = \hbar\omega_1\left(L_+ e^{-is\omega t} + L_- e^{+is\omega t}\right)$$

Exposure to Circularly Polarized Light

Effective Hamiltonian in Zero-Photon Sector:

$$\mathcal{H}_{\text{eff}}(\mathbf{k}) = \mathcal{P}\mathcal{H}_0(\mathbf{k})\mathcal{P} + \frac{\hbar^2\omega_1^2}{\Delta(\mathbf{k}) - \hbar\omega}\mathcal{P}L_+\mathcal{Q}L_-\mathcal{P} + \frac{\hbar^2\omega_1^2}{\Delta(\mathbf{k}) + \hbar\omega}\mathcal{P}L_-\mathcal{Q}L_+\mathcal{P}$$

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- Response is non-linear in input frequency

Exposure to Circularly Polarized Light

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- Response is non-linear in input frequency
- To produce a gap of 100 meV, one needs an input frequency in the range of TW/cm^2

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Future Work

- ✓ Coupling of light to itinerant moments

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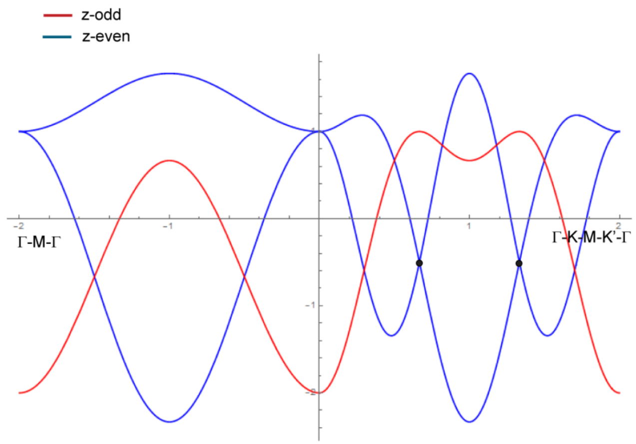
- ✓ Accounting for spin-orbit coupling
- ✓ Predicting material realization

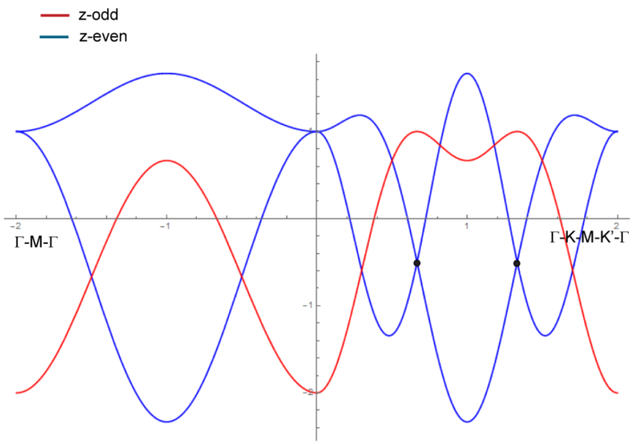
Conclusion

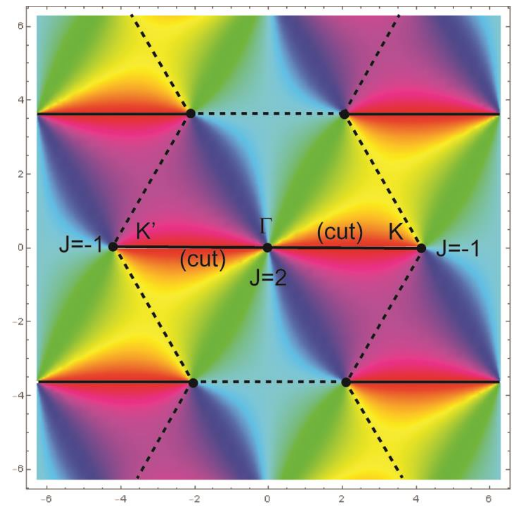
✓ The triangular lattice with *p* orbitals has protected line and point nodes



Conclusion

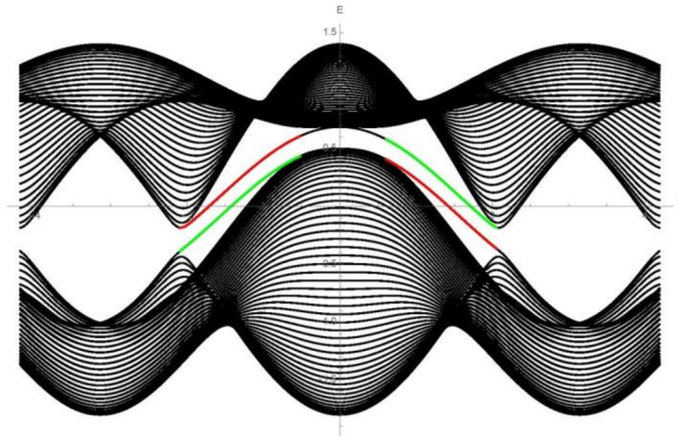
- ✓ The triangular lattice with *p* orbitals has protected line and point nodes
- ✓ The point nodes at the high-symmetry points carry compensating twists at high order

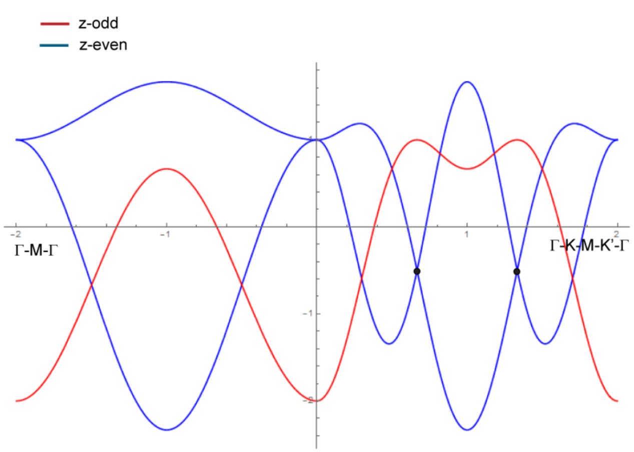


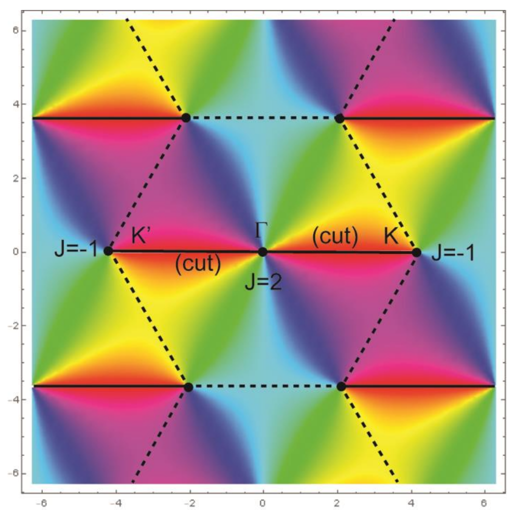


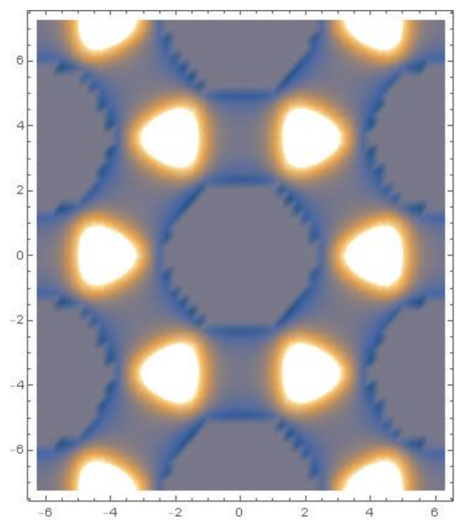
Conclusion

- ✓ The triangular lattice with *p* orbitals has protected line and point nodes
- ✓ The point nodes at the high-symmetry points carry compensating twists at high order
- ✓ Valley-symmetric mass gap produces an anomalous Hall effect



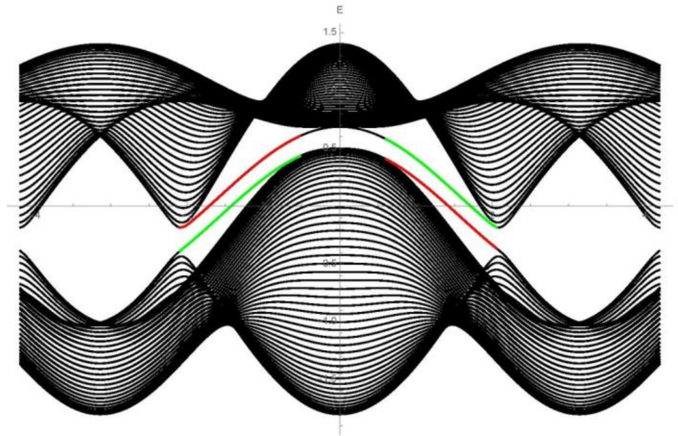


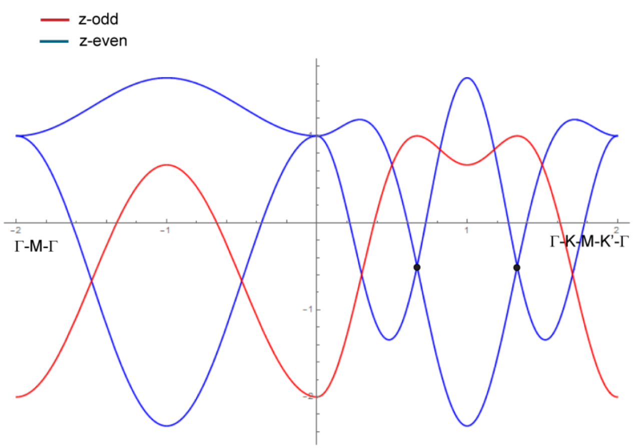


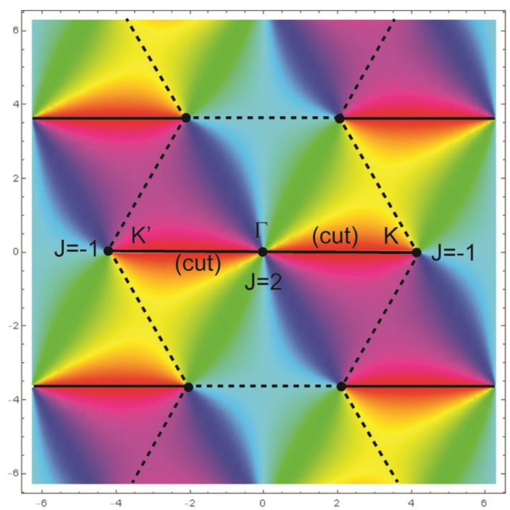


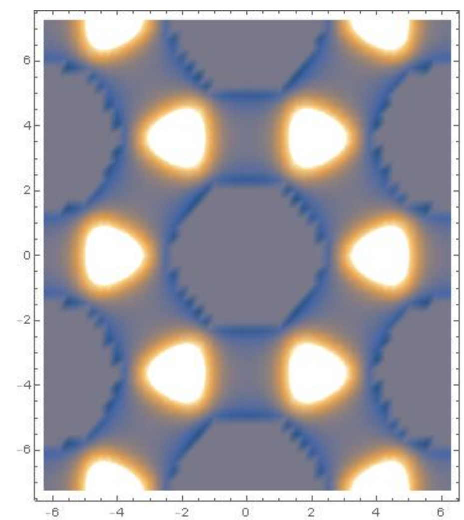
Conclusion

- ✓ The triangular lattice with *p* orbitals has protected line and point nodes
- ✓ The point nodes at the high-symmetry points carry compensating twists at high order
- ✓ Valley-symmetric mass gap produces an anomalous Hall effect
- ✓ Coupling to CPL produces such a nontrivial gap









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Thank **you** for the opportunity to present our work here in Việt Nam.