BIMETRIC THEORY OF FRACTIONAL QUANTUM HALL STATES

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PLAN

Introduction

- Fractional quantum Hall effect
- ``Non-relativistic'' Geometry
- Chern-Simons theory

Girvin-MacDonald-Platzman mode

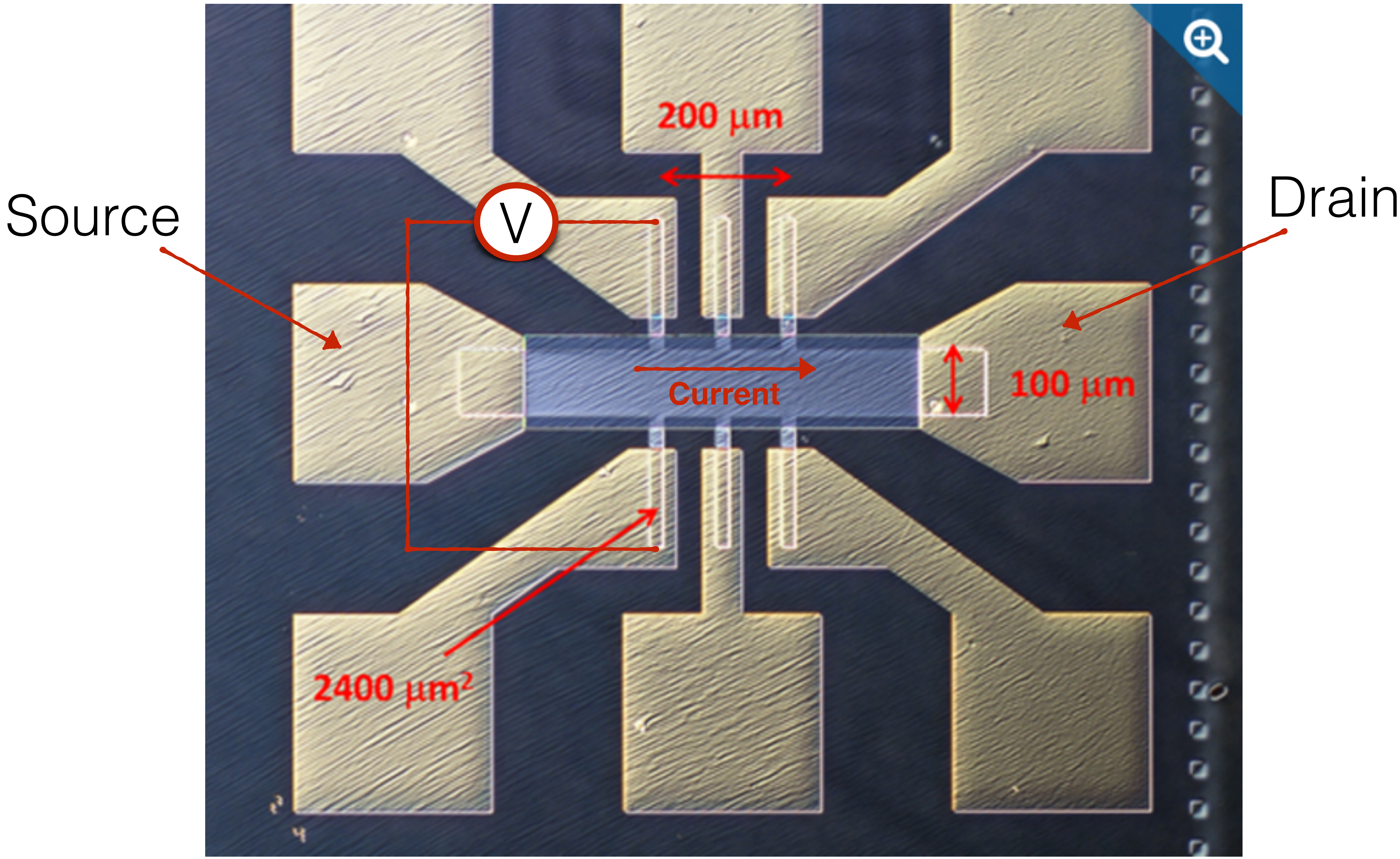
- Lowest Landau Level
- W_∞ algebra
- Single Mode Approximation

Bimetric theory of FQH states

- Bimetric theory
- How does it work ?
- Consistency checks
- Geometric quench

Conclusions and open directions

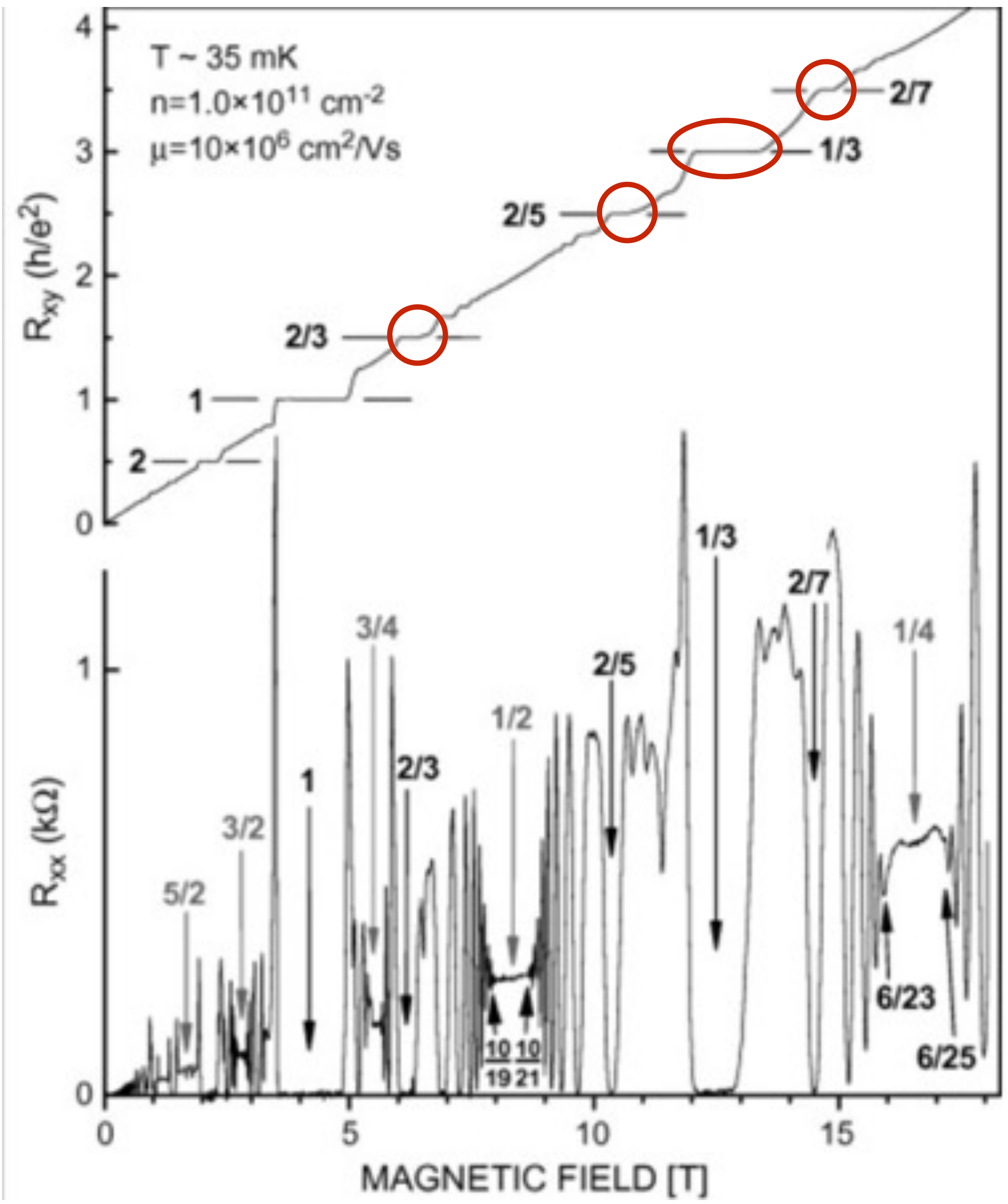
QUANTUM HALL BAR



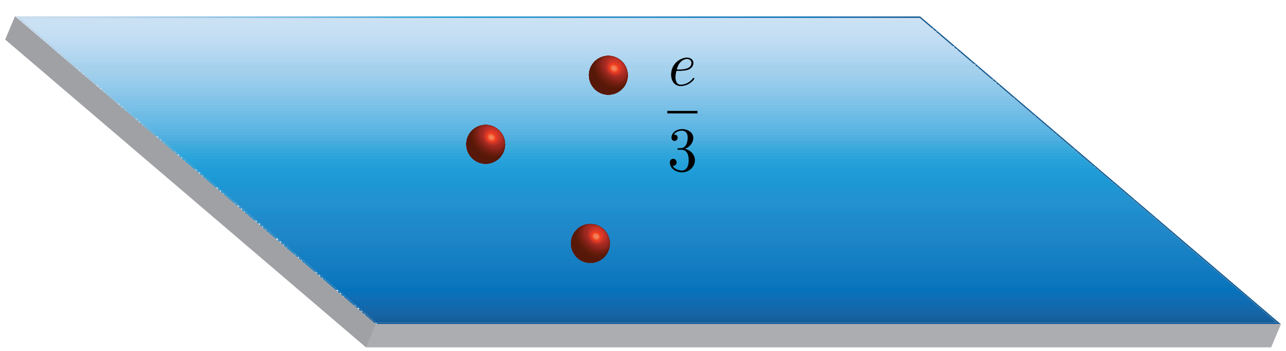
Gold-coated graphene quantum Hall bar,
Physical Measurement Laboratory (2014)

$$R_H = \frac{V}{I}$$

FRACTIONAL QUANTUM HALL EFFECT



$$H = \sum_{i,j} U(|x_i - x_j|)$$



- ⋆ Fractional Hall conductance
- ⋆ Fractionally charged quasiparticles
- ⋆ Fractional statistics
- ⋆ Topological degeneracy
- ⋆ …

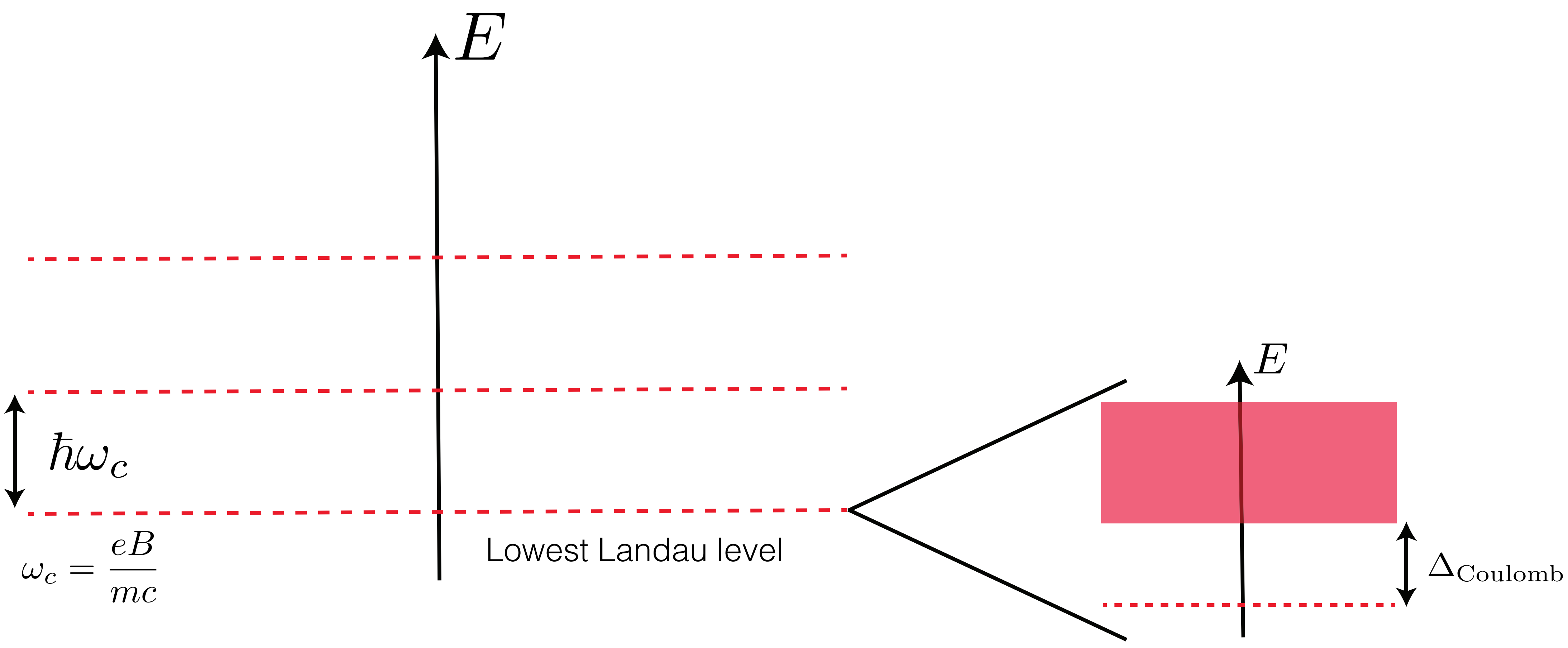
Laughlin 1983 Arovas, Schrieffer, Wilczek; Halperin 1984 Haldane, Rezayi 1985

AT THE PLATEAU

- Gap to all excitations (charged *and* neutral)
- All dissipative transport coefficients vanish
- Parity and time-reversal broken
- *No* Lorentz invariance
- Quantized non-dissipative transport coefficients
- *Not* uniquely characterized by the filling factor

$$N = \nu N_\phi \qquad\qquad \sigma_{xy} = \nu \frac{e^2}{h}$$

SPECTRUM OF THE FQH



Fractional quantum Hall problem is often treated in the **topological limit**

$$\hbar\omega_c \to \infty \qquad \Delta_{\text{Coulomb}} \to \infty$$

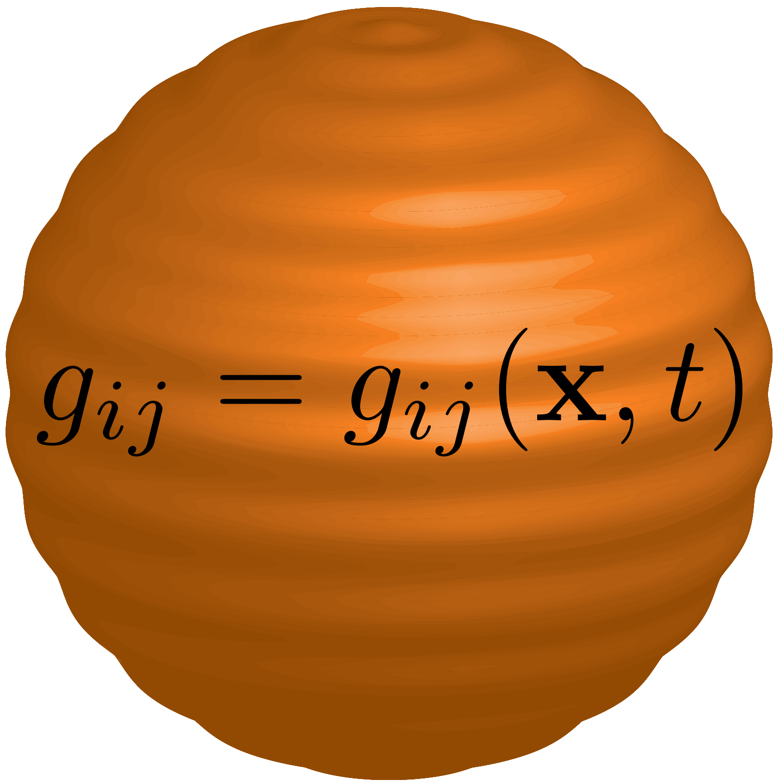
In this limit **Hamiltonian is 0** and dynamics occurs only at the edge

I would like to understand the FQH <u>away from the topological limit</u>

TOPOLOGICAL LIMIT

GEOMETRY

Geometry is encoded into time-dependent metric



$$ds^2 = g_{ij}(\mathbf{x}, t) dx^i dx^j$$

It is more convenient to use vielbeins

$$g_{ij} = e_i^A e_j^B \delta_{AB} \qquad \mathbf{g} = \mathbf{e} \cdot \mathbf{e}^T$$

There is a $SO(2)$ redundancy

Corresponding ``gauge field'' is the *spin* connection ω_μ

Spin connection is a ``vector potential'' for curvature

$$\frac{R}{2} = \partial_1 \omega_2 - \partial_2 \omega_1 \qquad \omega_0 \sim \epsilon_A{}^B e_B^i \partial_0 e_i^A$$

CHERN - SIMONS THEORY OF FQH STATES

Determines filling $\nu = k^{-1}$

electric charge of constituent particles

mean orbital spin

Wen-Zee term

$$S = \frac{k}{4\pi}\int ada - \frac{q}{2\pi}\int adA - \frac{s}{2\pi}\int ad\omega$$

quantum ``emergent'' gauge field

external e/m field

$SO(2)$ spin connection

Wen-Zee term *couples the TQFT to the geometry* of ambient space

Breaks Lorentz down to $SO(2)$ since $\omega_\mu = \omega_\mu^{AB}\epsilon_{AB}$

Wen, Zee 1991

WEN - ZEE TERM

Wen-Zee term couples the electron density to curvature

$$\rho = \frac{\nu}{2\pi} B + \frac{\nu s}{4\pi} R$$

Implies a global relation on a compact Riemann surface

$$N = \nu N_\phi + \nu \mathcal{S} \frac{\chi}{2}$$

← Euler characteristic

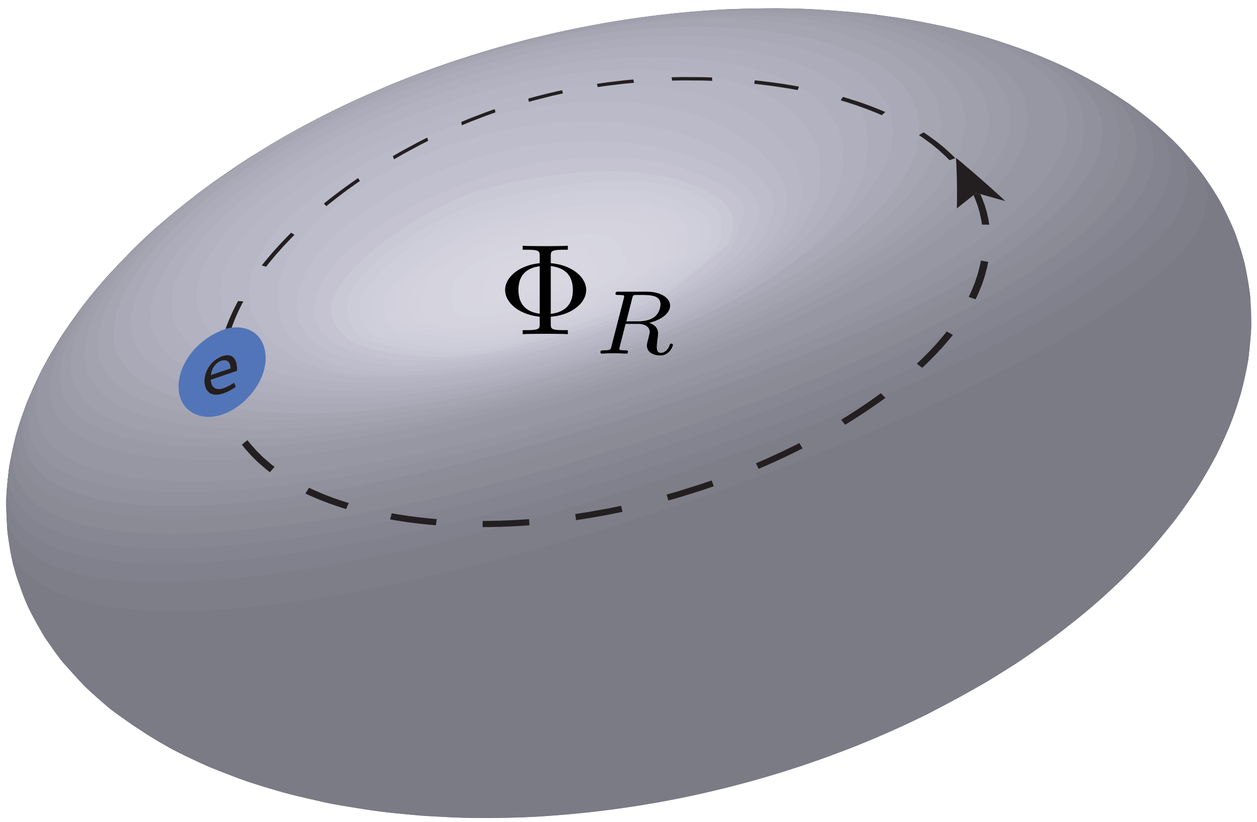
Quantum number $\mathcal{S} = 2s$ is called *Shift*

Also describes the quantum Hall viscosity

$$\langle T_{xx} T_{xy} \rangle = i\omega \eta_H \qquad \eta_H = \hbar \frac{\mathcal{S}}{4} \bar{\rho}$$

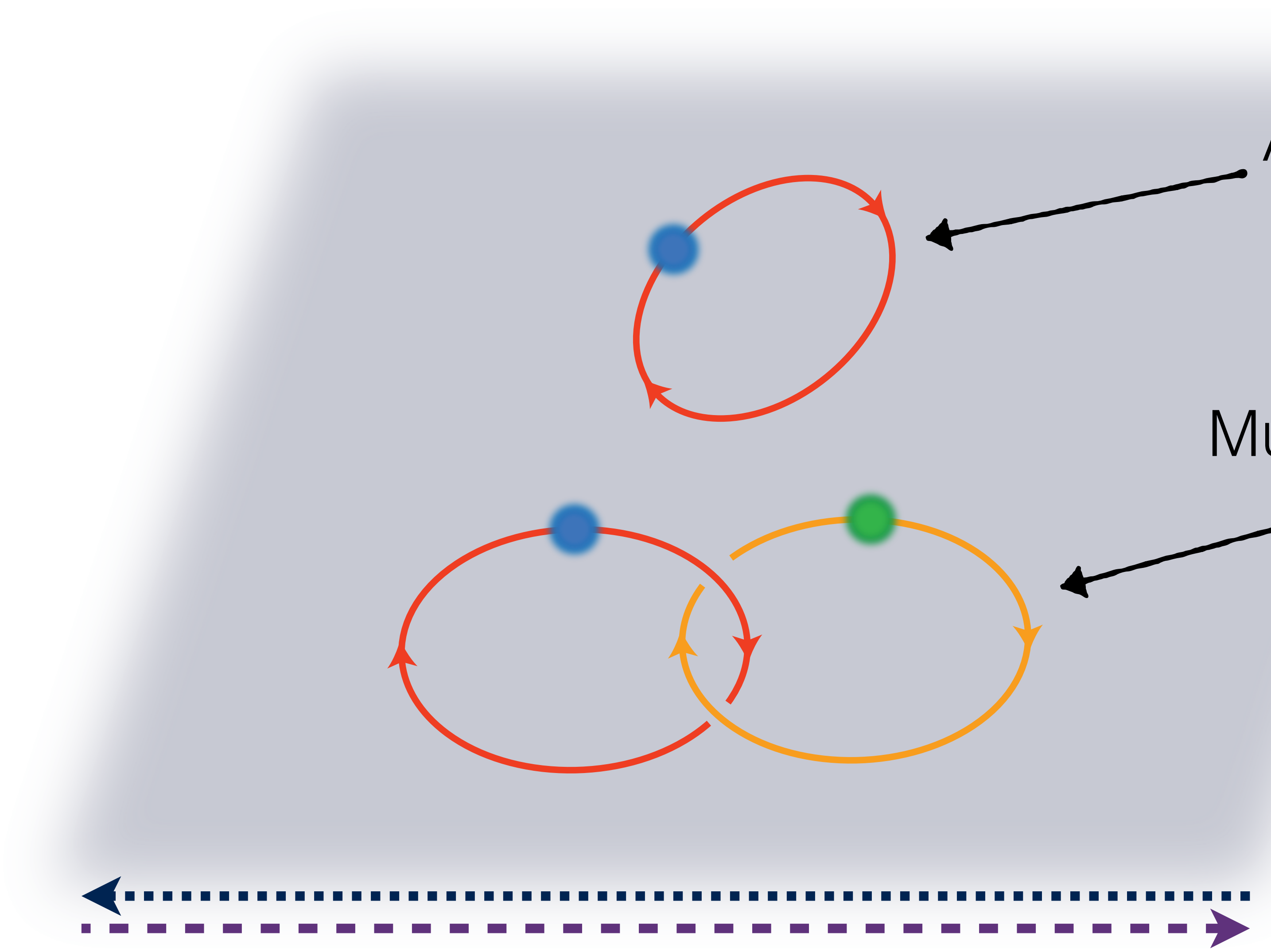
Haldane 1983 Wen, Zee 1991 Avron Zograf Seiler 1995 Read 2009

``GRAVITATIONAL'' AHARONOV - BOHM EFFECT



$$\Psi \longrightarrow e^{2\pi i S \Phi_R} \Psi$$

Wen Zee 1992



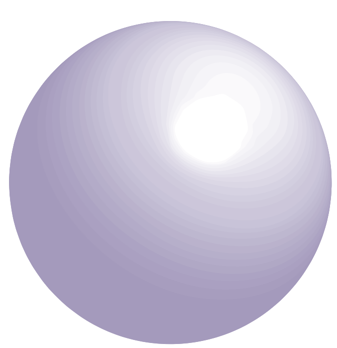
Aharonov-Bohm phases

Mutual statistics*

Chiral edge modes: tunneling exponents, thermal Hall conductance

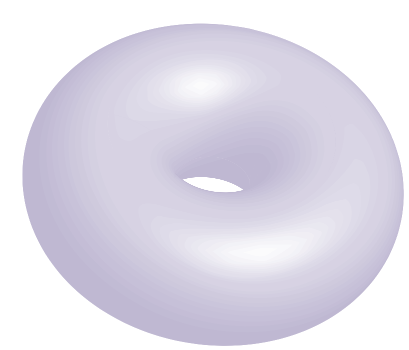
$$W[A,\omega] = \int \mathcal{D}a\, e^{iS[a;A,\omega]}$$

→ Linear response: Hall conductance, Hall viscosity,…



Shift $\mathcal{S} = 2\bar{s}$

$$N = \nu N_\phi + \nu \mathcal{S}$$



Ground state degeneracy k

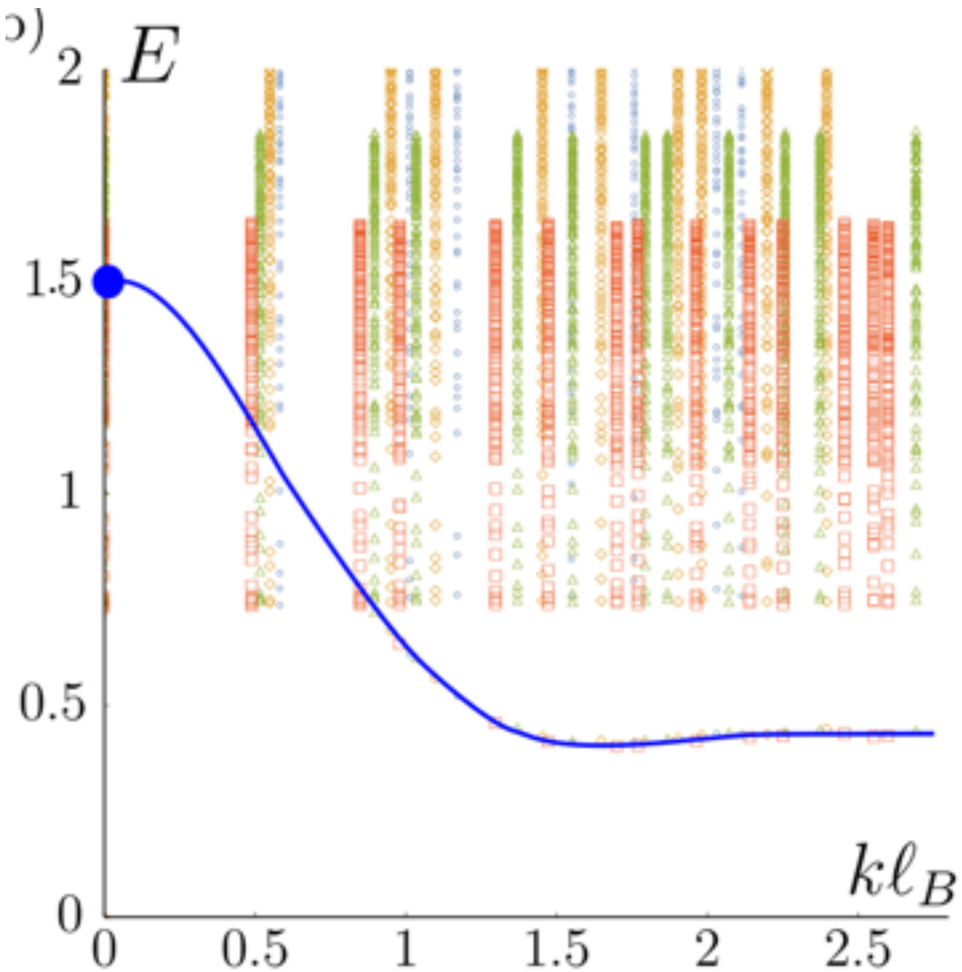
*after AB phases are subtracted

BEYOND TQFT (THEORETICAL) TOOLS

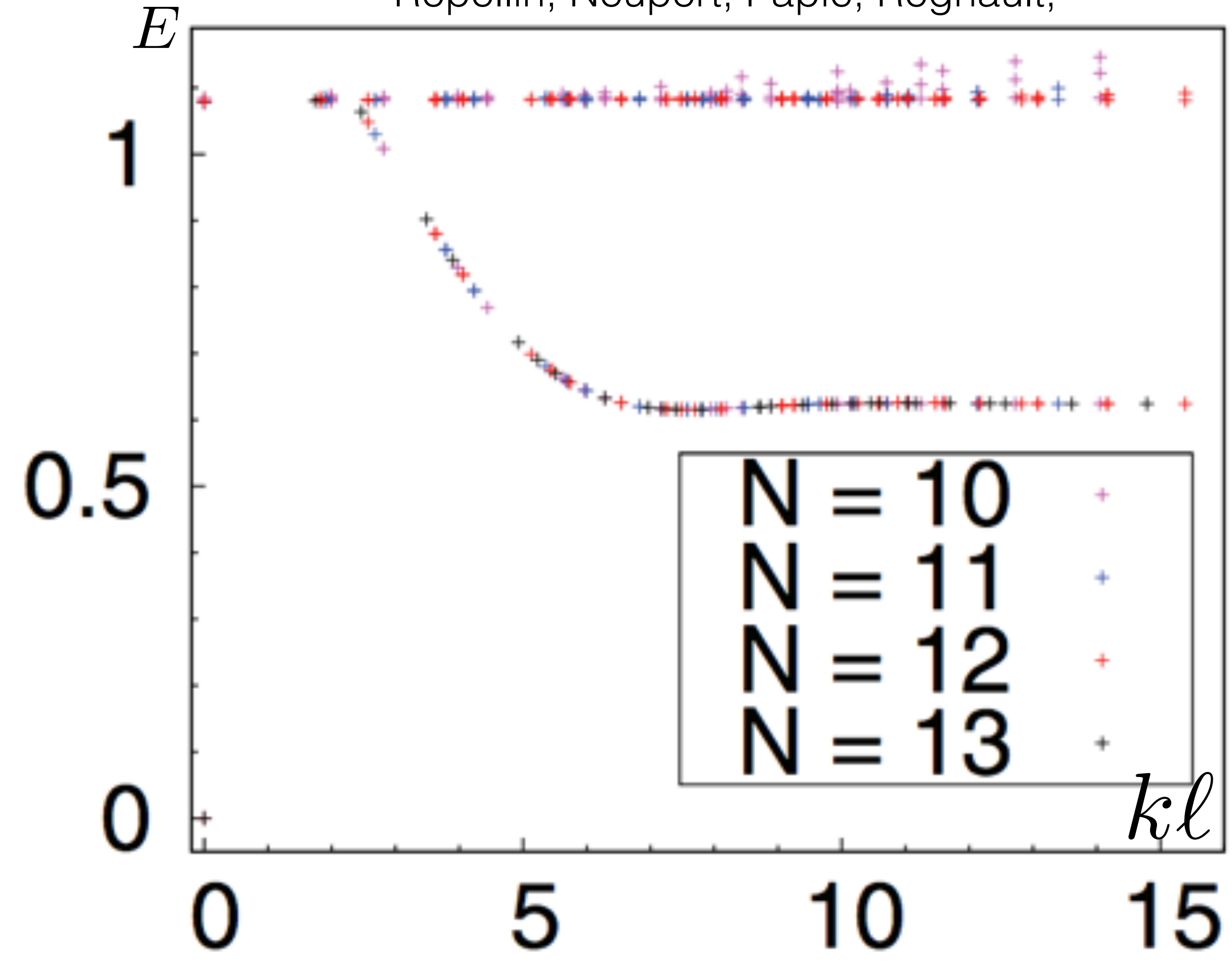
Beyond TQFT we face a strongly interacting problem

- Trial states
- Flux attachment (composite bosons and fermions)
- Exact diagonalization
- *Bimetric theory*

SPECTRUM OF FQH PROBLEM



Repellin, Neupert, Papić, Regnault,



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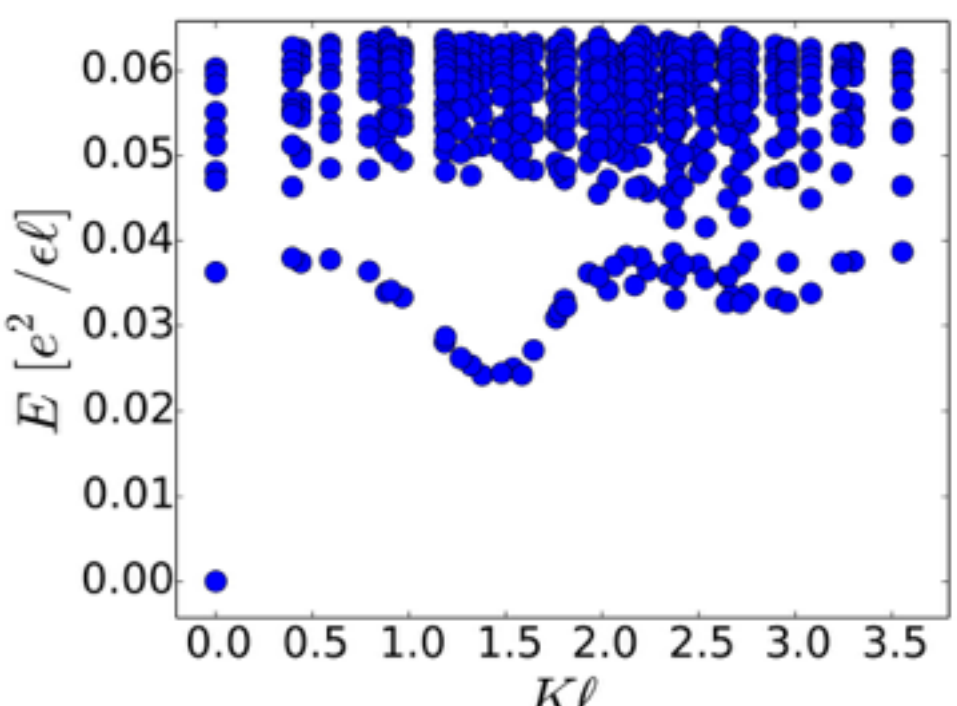
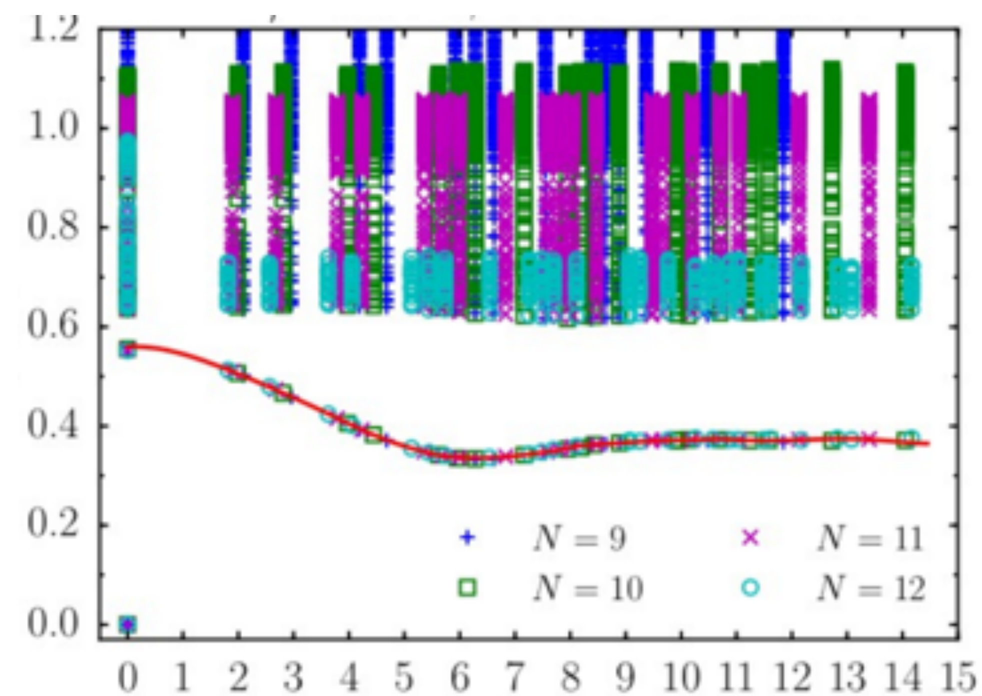


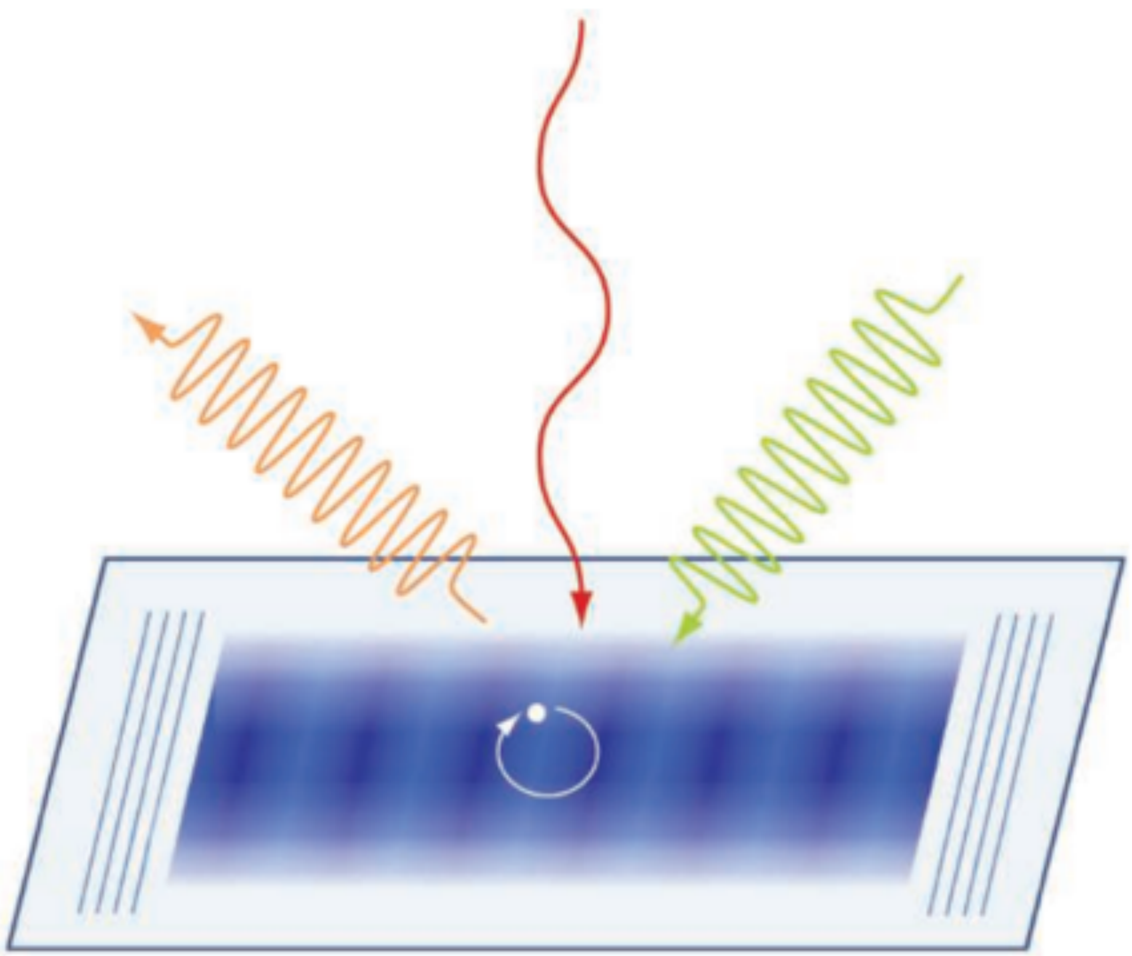


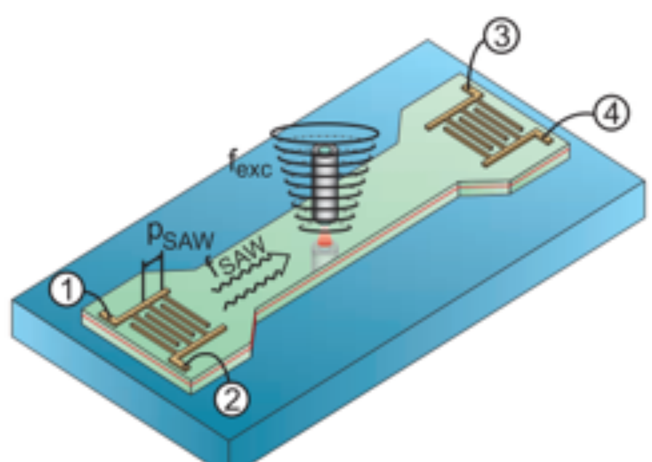
Haldane, Rezayi 1985

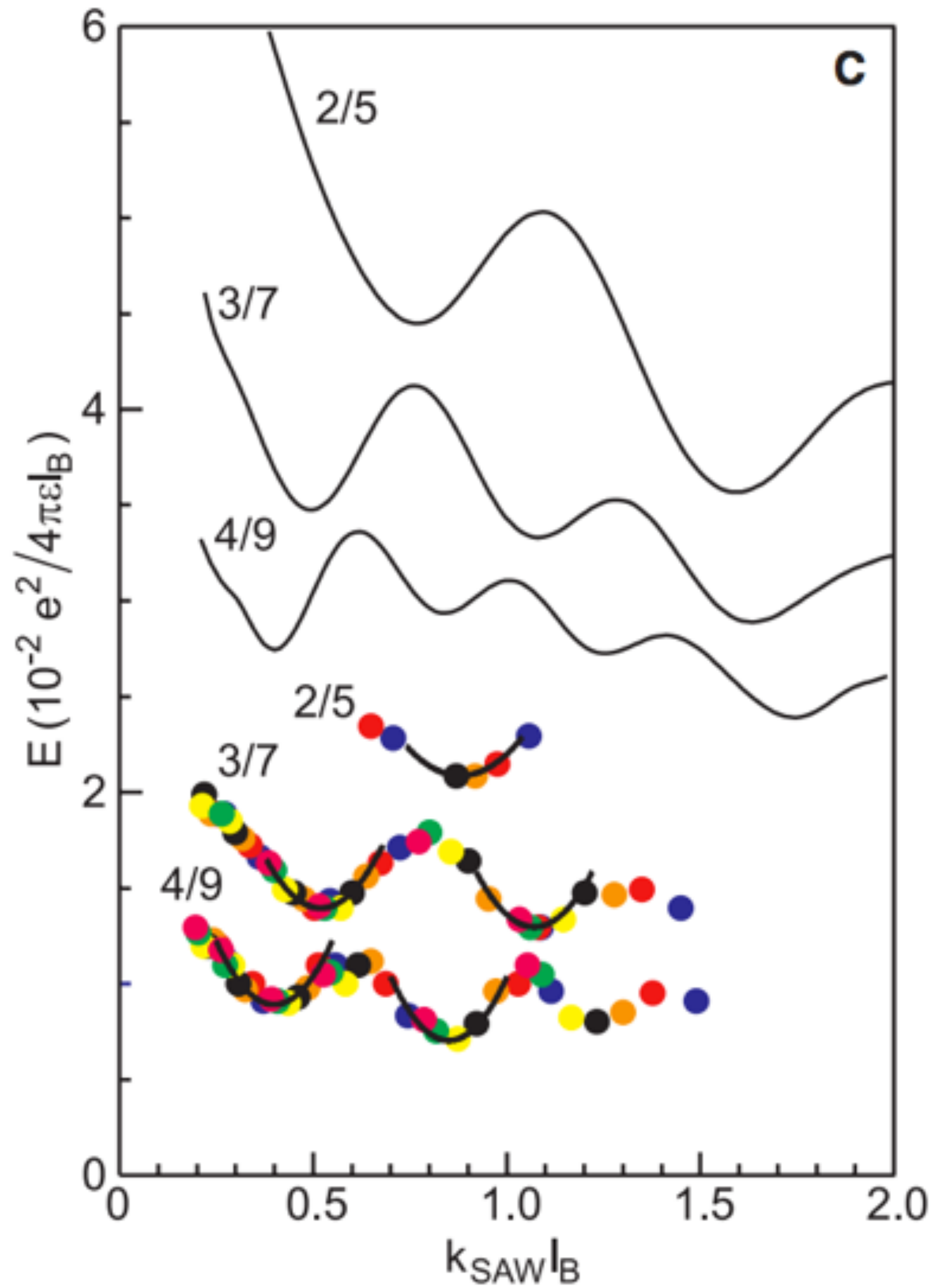
Girvin, MacDonald, Platzman, 1985

At $E \sim gap$ there is a collective mode

OBSERVATION OF THE GMP MODE







Kang, Pinczuk, Dennis, Pfeiffer, West 2001

Kukushkin, Smet, Scarola, Umansky, von Klitzing 2009

GIRVIN - MACDONALD - PLATZMAN MODE

The electron density operator $z_j = x_j + iy_j$ is the electron coordinate

$$\rho(\mathbf{k}) = \sum_{j=1}^{N_{\rm el}} e^{i\frac{\mathrm{k}\bar{z}_j + \bar{\mathrm{k}} z_j}{2}} \xrightarrow[\bar{z} \to 2\partial_z]{\text{LLL projection}} \bar{\rho}(\mathbf{k}) = \sum_{j=1}^{N_{\rm el}} e^{i\mathrm{k}\partial_{z_j}} e^{i\frac{\bar{\mathrm{k}} z_j}{2}}$$

Projected densities **do not commute**, instead they form a W_∞ algebra

$$[\bar{\rho}(\mathbf{k}), \bar{\rho}(\mathbf{q})] = 2i\sin\left[\frac{\ell^2}{2}\mathbf{k}\times\mathbf{q}\right]\bar{\rho}(\mathbf{k}+\mathbf{q})$$

This algebra is believed to be at the heart of the Lowest Landau level problem

GMP have argued that the collective mode is a projected density wave

$$|\mathbf{k}\rangle = \bar{\rho}(\mathbf{k})|0\rangle$$

Girvin, MacDonald, Platzman, 1985

GIRVIN - MACDONALD - PLATZMAN MODE II

At long wavelengths the GMP state takes form

$$|\mathbf{k}\rangle \approx \left[k^2 T^+ + \bar{k}^2 T^- + \dots\right]|0\rangle$$

The spin-2 operators $T^\pm$ form an $sl(2,\mathbb{R})$ algebra

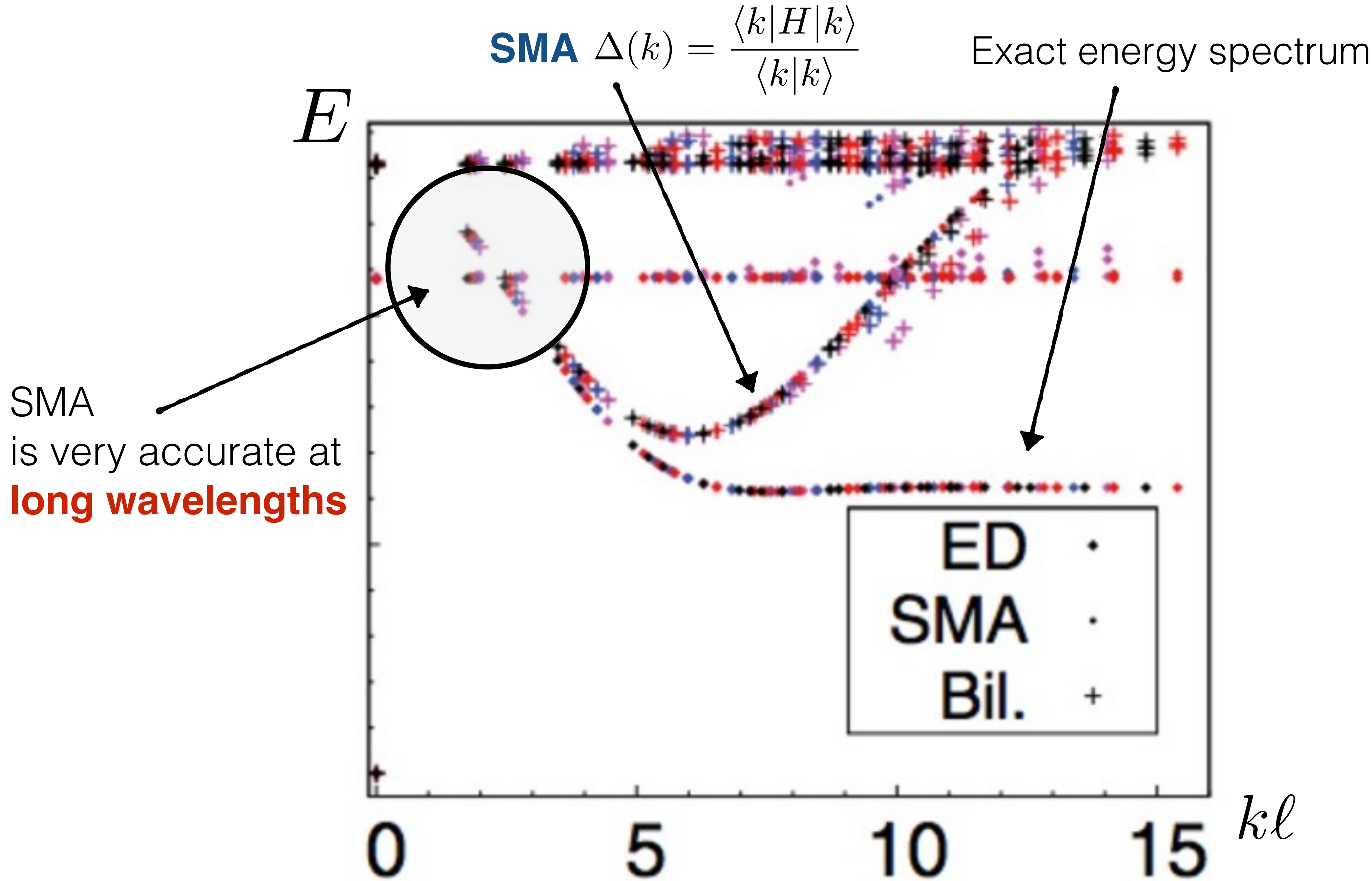
$$\boxed{\left[T^+, T^-\right] = 2L} \qquad \boxed{\left[T^\pm, L\right] = \pm T^\pm}$$

Angular momentum

In the LLL $T^\pm$ are differential operators

$$T^+ \propto \sum_i z_i \qquad T^- \propto \sum_i \frac{\partial}{\partial z_i}$$

HOW GOOD IS SMA ?



Repellin, Neupert, Papić, Regnault

We will construct a long wavelength effective theory of GMP mode

GENERAL REMARKS ABOUT THE GMP MODE

★ Universally present in **fractional** QH states

★ Absent in **integer** QH states

★ Angular momentum or ``spin'' **2**, regardless of microscopic details

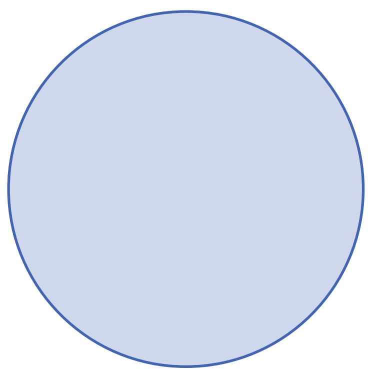
★ Effective theory of the GMP mode should to be a *theory of a massive spin-2 excitation*

BIMETRIC THEORY

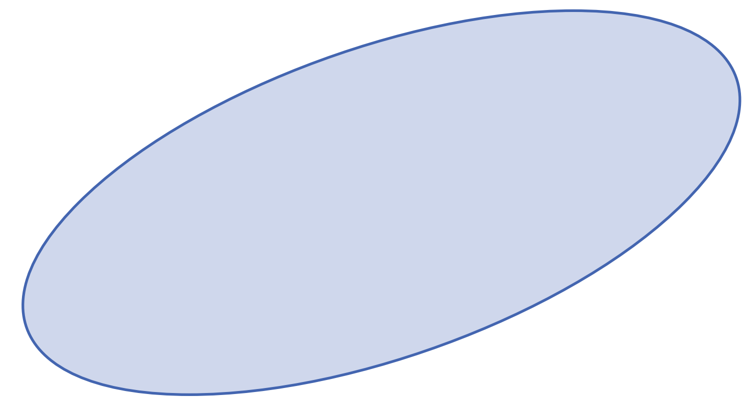
SPIN-2 DEGREE OF FREEDOM

A spin-2 mode is described by a symmetric 2x2 matrix $\hat{\mathrm{g}}_{ij}$ with $\det \hat{\mathrm{g}}_{ij} = 1$

Can be visualized as a solution of equation $\hat{\mathrm{g}}_{ij} x^i x^j = 1$

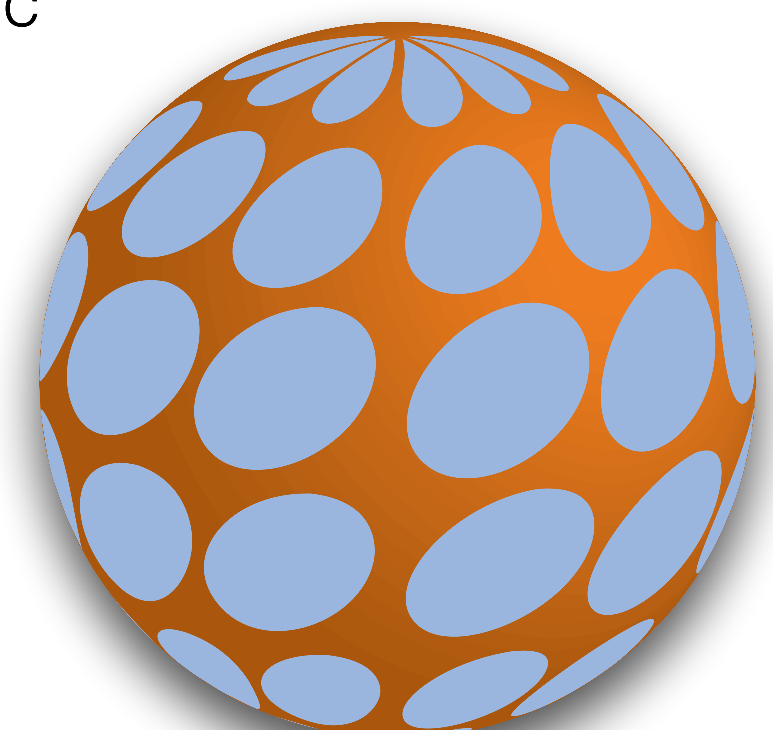


$\hat{\mathrm{g}}_{ij} = \delta_{ij}$



$\hat{\mathrm{g}}_{ij}$

Together with the ambient metric



BIMETRIC GEOMETRY

The spin-2 mode is described by a symmetric tensor $\hat{\mathrm{g}}_{ij}(\mathbf{x}, t)$

Subject to a constraint $\det \hat{\mathrm{g}}_{ij} = \det \mathrm{g}_{ij}$

Introduce the ``vielbein'' $\hat{\mathrm{g}}_{ij} = \delta_{\alpha\beta} \hat{e}^{\alpha}_{i} \hat{e}^{\beta}_{j}$

$\widehat{SO}(2)$ spin connection and curvature follow

$$\frac{\hat{R}}{2} = \partial_1 \hat{\omega}_2 - \partial_2 \hat{\omega}_1 \qquad \hat{\omega}_0 = \frac{1}{2} \epsilon^{\alpha}{}_{\beta} \hat{e}^{i}_{\alpha} \partial_0 \hat{e}^{\beta}_{i}$$

Not the same as two copies of Riemannian geometry

$$\mathrm{Diff} \times \widehat{\mathrm{Diff}} \to \mathrm{Diff}_{\mathrm{diag}}$$

This geometry involves **two** metrics $\hat{\mathrm{g}}_{ij}, \mathrm{g}_{ij}$, hence *bimetric**

*Re-appeared recently in theories of massive gravity de Rham, Gabadadze, Tolley 2010

BIMETRIC THEORY

Chern-Simons theory interacting with fluctuating metric $\hat{g}_{ij}(\mathbf{x}, t)$

Pronounced: ``sigma''

$$\mathcal{L} = \frac{k}{4\pi} ada - \frac{1}{2\pi} Ada - \frac{s}{2\pi} ad\omega - \frac{\varsigma}{2\pi} ad\hat{\omega} - \mathcal{H}[\hat{g}; g]$$

Hamiltonian

$$\mathcal{H} = \frac{m}{2}(\hat{g}_{ij} g^{ij} - \gamma)^2$$

Integrate out the internal gauge field a

$$\mathcal{L} = \mathcal{L}_1[A, g] + \mathcal{L}_{bm}[\hat{g}; A, g]$$

Where $\mathcal{L}_1[A, g]$ contains no dynamics and is discarded

For IQH $k = 1$ **there is no *intra-LL dynamics***

$$\mathcal{L} = \mathcal{L}_1[A; g]$$

AG, Son 2017

BIMETRIC THEORY

Effective theory of the spin-2 mode at long wavelengths

$$\mathcal{L} = \frac{\nu\varsigma}{2\pi}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu\hat{\omega}_\rho - \frac{m}{2}\left(\hat{\mathrm{g}}_{ij}\mathrm{g}^{ij} - \gamma\right)^2$$

$p\dot{q}$ — type term

Hamiltonian

ς quantized phenomenological parameter

m the energy scale of the spin-2 mode

γ phenomenological parameter responsible for the nematic transition

AG, Son 2017

GEOMETRIC OPERATORS

Density and current operators acquire geometric meaning

Fluctuations of electron density = fluctuations of local Ricci curvature

$$\rho = \frac{\nu\varsigma}{4\pi}\hat{R}$$

Fluctuations of electron current = fluctuations of ``gravi-electric'' field

$$j^i = \frac{\nu\varsigma}{2\pi}\epsilon^{ik}\hat{\mathcal{E}}_k$$

To the leading order in $\mathbf{k}$, *everything* is determined by ς

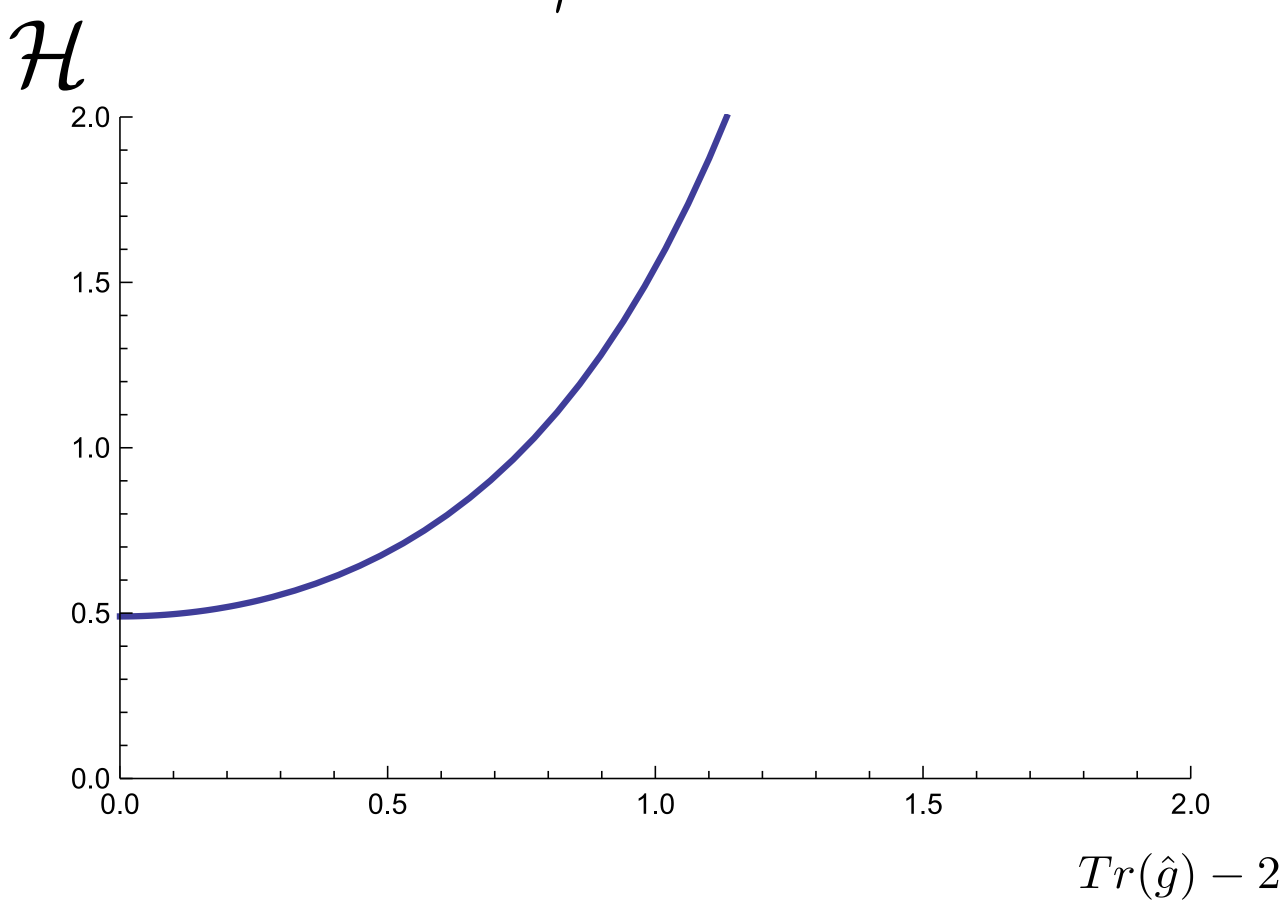
Continuity equation holds identically

$$\partial_0\hat{R} + \epsilon^{ik}\partial_i\hat{\mathcal{E}}_k \equiv 0$$

POTENTIAL TERM

$$\mathcal{H}[\hat{g}_{ij}] = \frac{M}{2}\left(\frac{Tr(\hat{g})}{2} - \gamma\right)^2$$

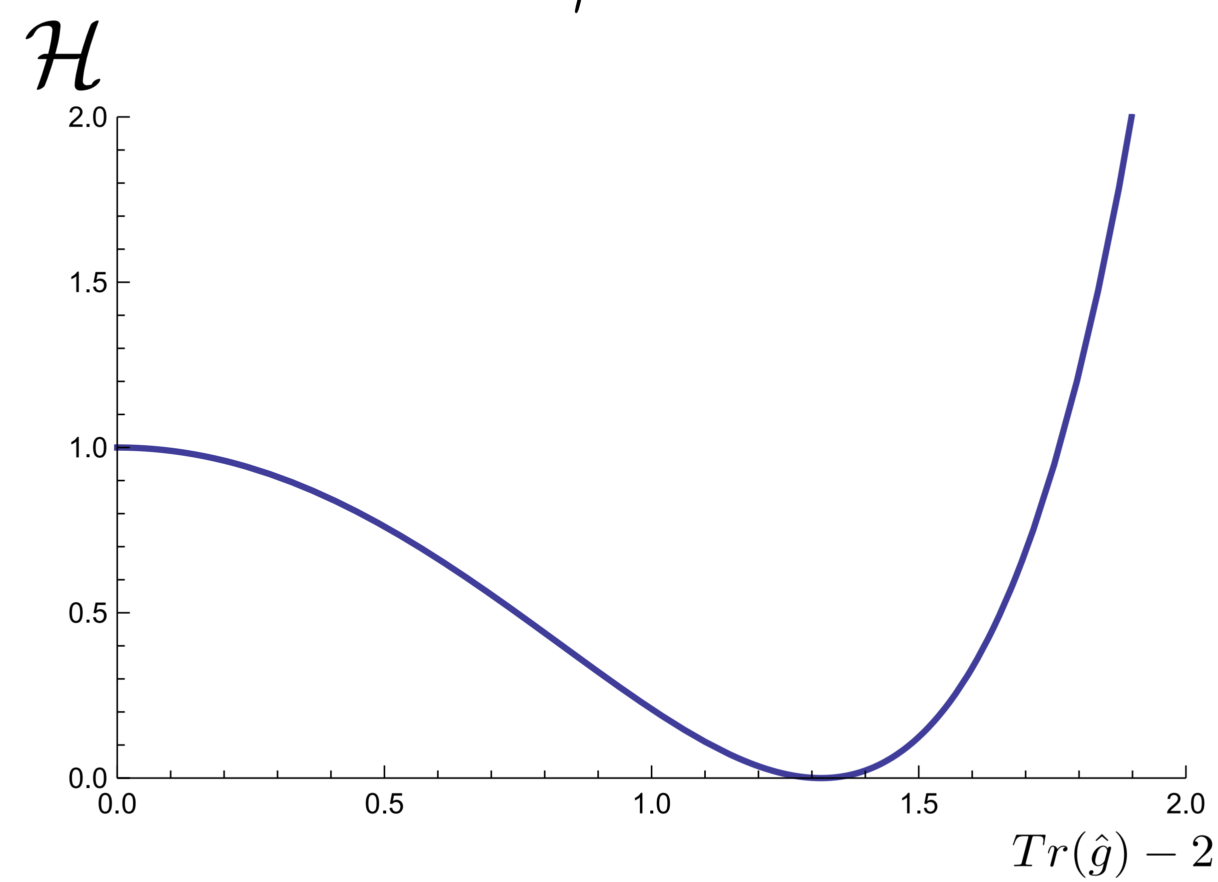
$\gamma < 1$



gapped ``symmetric'' phase

$$\hat{g}_{ij} = \delta_{ij}$$

$\gamma > 1$



gapless nematic phase

$$\hat{g}_{ij} = \hat{g}^{(0)}_{ij}(\gamma) \neq \delta_{ij}$$

LINEARIZATION

In flat space we chose the parametrization

$$\hat{g}_{ij} = \exp\begin{pmatrix} Q_2 & Q_1 \\ Q_1 & -Q_2 \end{pmatrix} \qquad \begin{aligned} Q &= Q_1 + iQ_2 \\ \bar{Q} &= Q^* \end{aligned}$$

and linearize around isotropic configuration

$$\hat{g}_{ij} = \delta_{ij} \qquad Q = 0$$

to find

Gap of the GMP mode

$$\mathcal{L}_{\rm bm} \approx i\frac{\varsigma\bar{\rho}}{4}\bar{Q}\dot{Q} - \frac{m(1-\gamma)}{2}|Q|^2$$

Maciejko, Hsu, Kivelson, Park, Sondhi 2013

You, Cho, Fradkin 2014

ELECTRON DENSITY AND CURRENT

Fluctuations of electron density are governed by fluctuations of local Ricci curvature

$$\delta\rho = \frac{\nu\varsigma}{4\pi}\hat{R}$$

Projected static structure factor computation reproduces a classic result

$$\bar{s}(k) = 2\pi\nu^{-1}\langle\delta\rho_{-k}\delta\rho_k\rangle = \frac{2|\varsigma|}{8}|k|^4 + \dots$$

This is to be compared with the general result

$$\bar{s}(k) = \frac{|\mathcal{S}-1|}{8}|k|^4 + \dots$$

Leading to identification

$$2|\varsigma| = |\mathcal{S}-1|$$

Vanishes for IQH

AG, Son 2017

CANONICAL QUANTIZATION

Turn off electric field

$$\frac{\nu\varsigma}{2\pi} A d\hat{\omega} = \frac{\nu\varsigma}{2\pi} \bar{B}\hat{\omega}_0 = \varsigma\bar{\rho}\epsilon_\alpha{}^\beta \hat{e}^i_\beta \frac{\partial}{\partial t} \hat{e}^\alpha_i$$

From the action we read the commutation relations

$$\left[\hat{e}^i_\alpha(\mathbf{x}), \hat{e}^\beta_j(\mathbf{x}')\right] = -\frac{2i}{\varsigma\bar{\rho}} \delta^i_j \epsilon_\alpha{}^\beta \delta(\mathbf{x} - \mathbf{x}')$$

Haldane 2011

Appeared in Verlinde 1989

GMP ALGEBRA

Components of intrinsic metric do not commute with each other

$$\left[\hat{\mathrm{g}}_{zz}(\mathbf{x}), \hat{\mathrm{g}}_{\bar{z}\bar{z}}(\mathbf{x}')\right] = 2\hat{\mathrm{g}}_{z\bar{z}}(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}')$$

$$\left[\hat{\mathrm{g}}_{zz}(\mathbf{x}), \hat{\mathrm{g}}_{z\bar{z}}(\mathbf{x}')\right] = \hat{\mathrm{g}}_{zz}(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}')$$

$$\left[\hat{\mathrm{g}}_{\bar{z}\bar{z}}(\mathbf{x}), \hat{\mathrm{g}}_{z\bar{z}}(\mathbf{x}')\right] = -\hat{\mathrm{g}}_{\bar{z}\bar{z}}(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}')$$

These relations define $\mathfrak{sl}(2,\mathbb{R})$ Lie algebra

The density operators form the w_∞ algebra = small $\mathbf{k}$ limit of the GMP

$$[\delta\rho(\mathbf{k}), \delta\rho(\mathbf{q})] = i\ell^2(\mathbf{k}\times\mathbf{q})\delta\rho(\mathbf{k}+\mathbf{q})$$

Bimetric theory reproduces essential FQH features beyond topological order

AG, Son 2017

GENERAL STRUCTURE

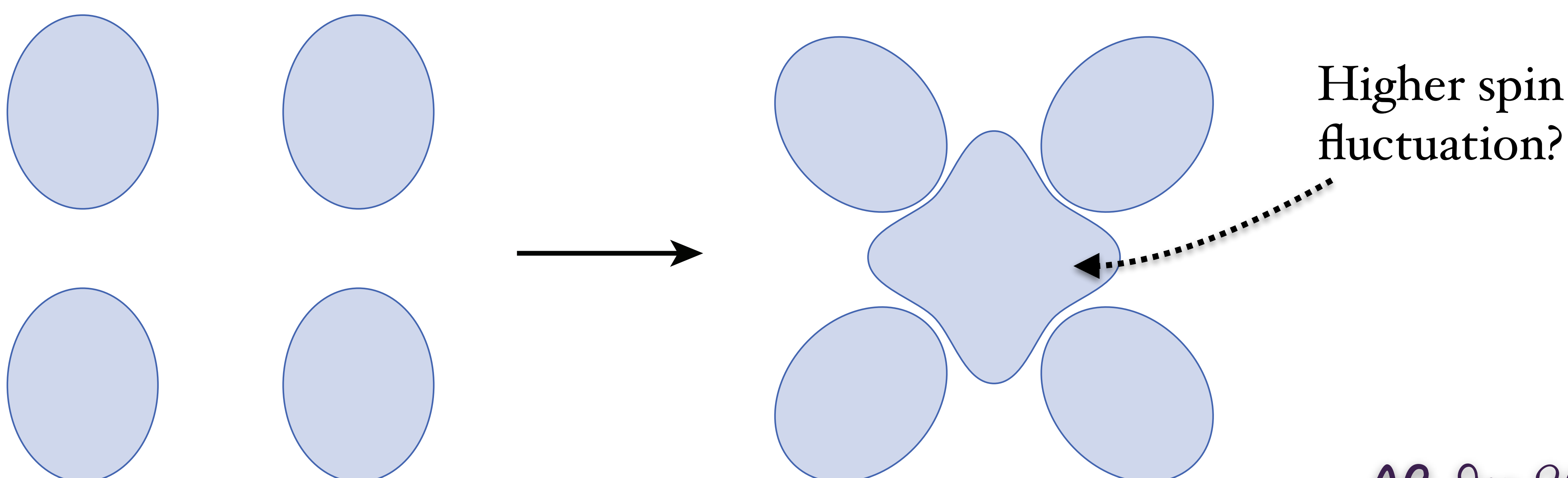
$$\mathcal{L} = \frac{\nu\varsigma}{2\pi} A d\hat{\omega} - \frac{\hat{c}}{4\pi}\hat{\omega} d\hat{\omega} - \mathcal{H}[\hat{\mathrm{g}}_{ij}; \mathrm{g}_{ij}]$$

$p\dot{q}$ — type terms

Non-universal Hamiltonian, depends on interactions

Gravitational Chern-Simons term breaks $\mathfrak{sl}(2,\mathbb{R})$ but respects w_∞ for density

This term turns on when the metric $\hat{\mathrm{g}}_{ij}$ is inhomogeneous



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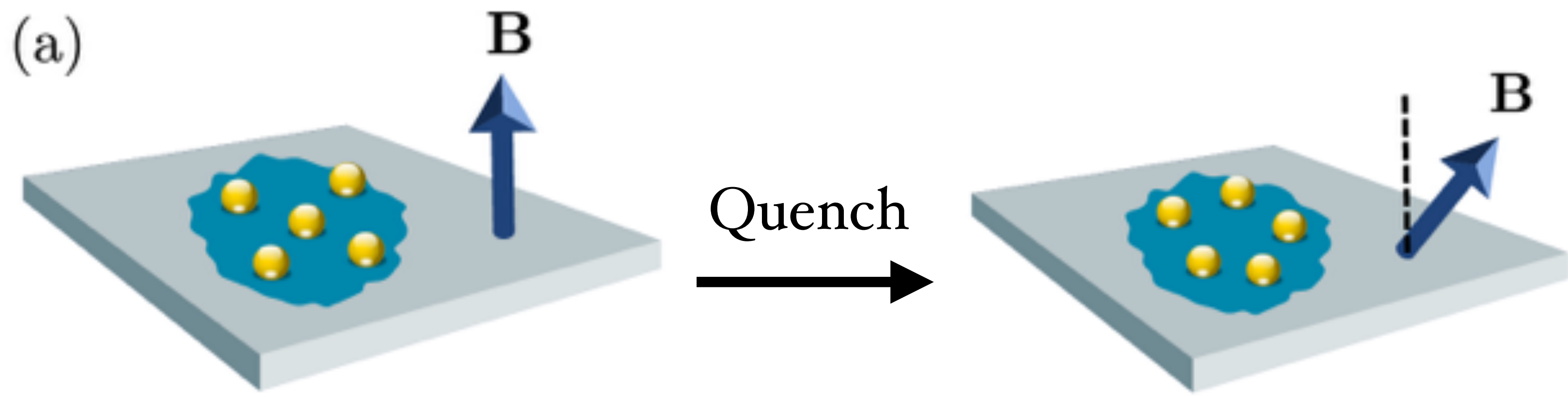
SUMMARY

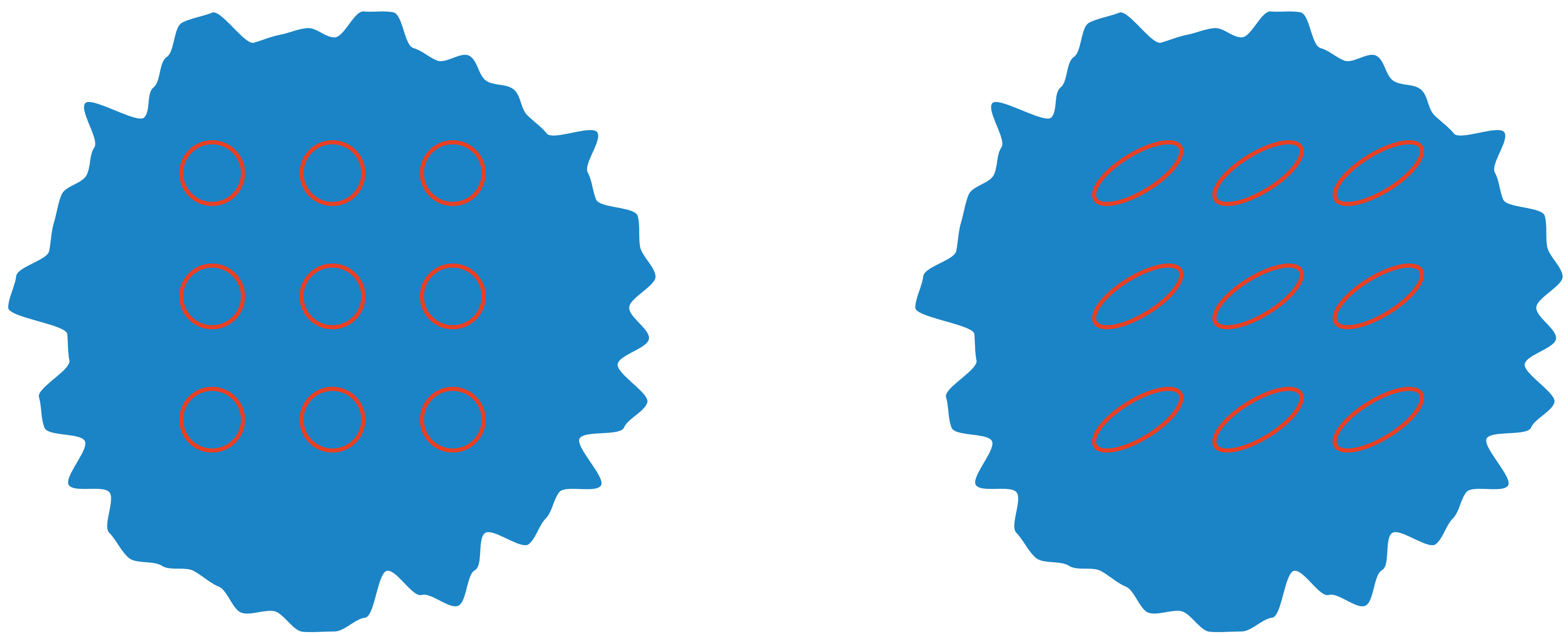
- ⋆ *Projected* static structure factor up to $|\mathbf{k}|^6$
- ⋆ Dispersion relation of the spin-2 mode up to $|\mathbf{k}|^2$
- ⋆ *Absence* of the spin-2 mode
- ⋆ Manifest Particle-Hole duality
- ⋆ Girvin-MacDonald-Platzman algebra holds up to $|\mathbf{k}|^4$
- ⋆ DC Hall conductivity to $|\mathbf{k}|^2$
- ⋆ Hall viscosity to $|\mathbf{k}|^2$
- ⋆ Hints at rich structure of the full W_∞ theory
- ⋆ Agrees with Dirac theory for Jain states close to 1/2

Nguyen, **AG**, Son 2017

AG, Son 2017

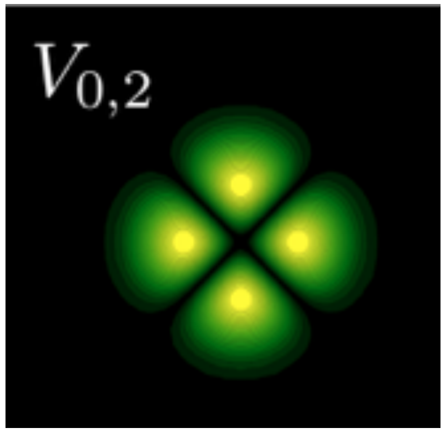
PROBING THE GEOMETRIC DEGREE OF FREEDOM





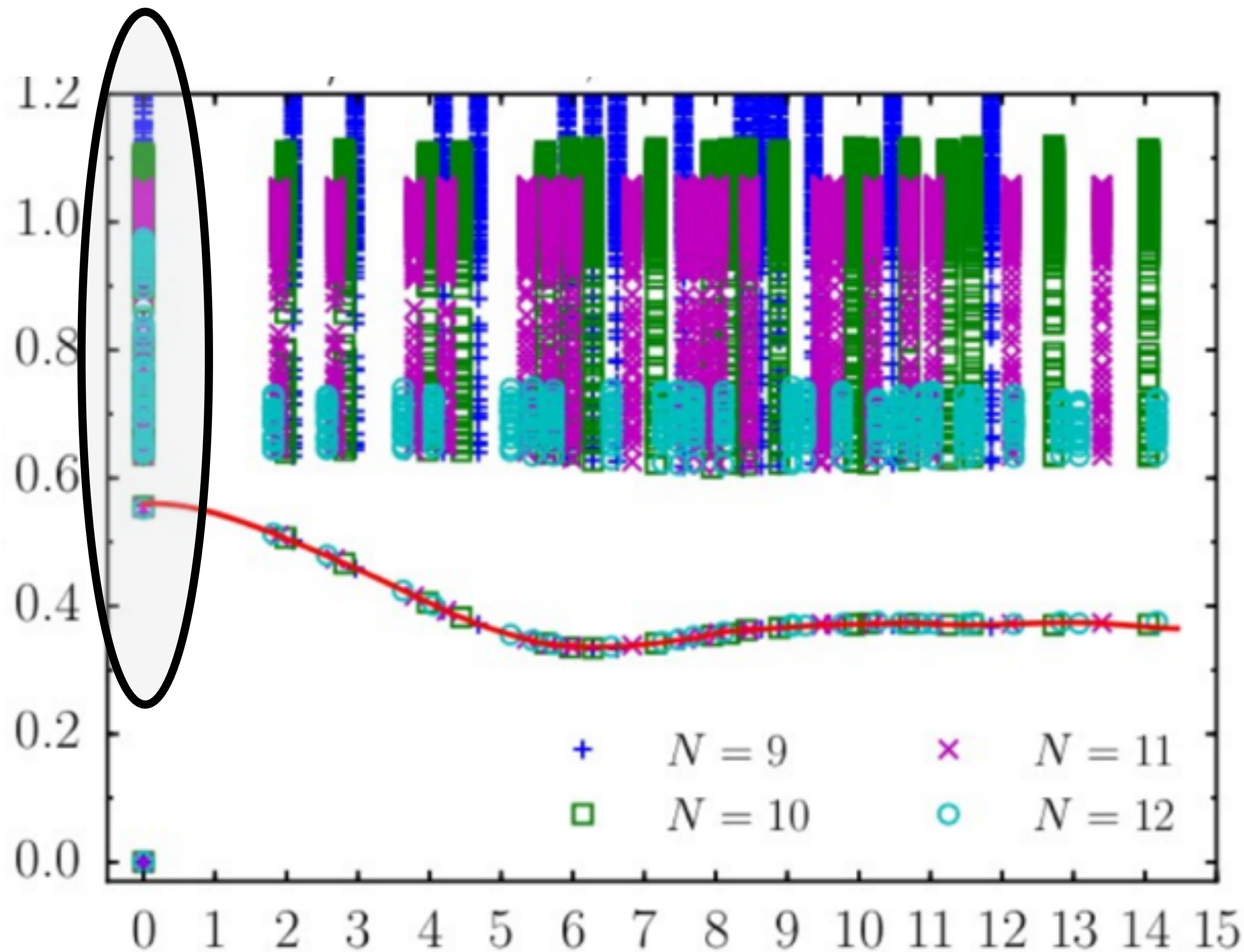
Quantitatively, we suddenly switch on a spin-2 perturbation

$$\mathcal{H} \longrightarrow \mathcal{H} + V_{0,2}$$

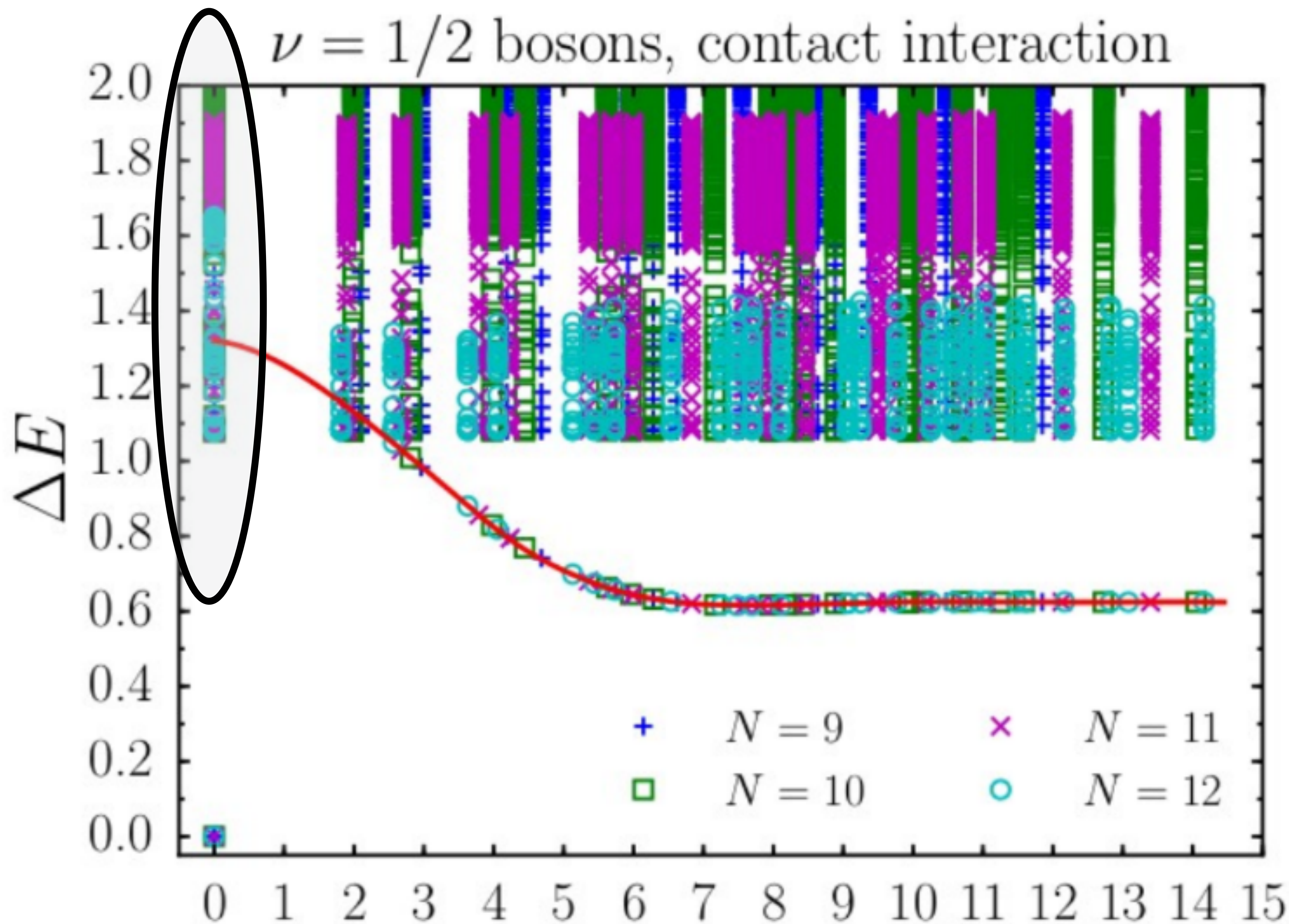


Liu, **AG**, Papic (2018)

GLOBAL QUENCH



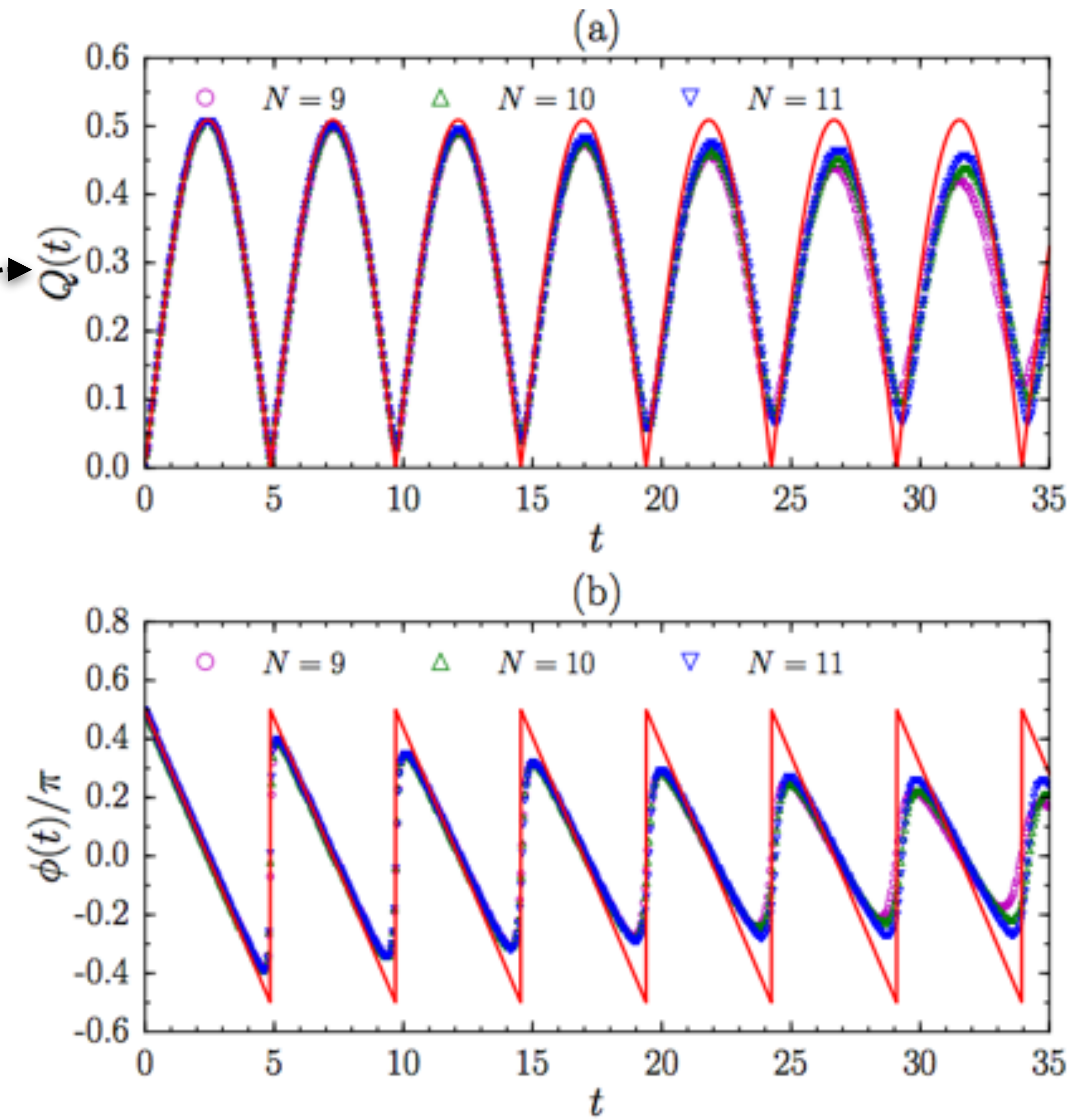
GLOBAL QUENCH



PROBING THE GEOMETRIC DEGREE OF FREEDOM

Dynamics is obtained via solving the EoM of bimetric theory in tilted magnetic field

$$\hat{g}_{ij} = \begin{pmatrix} \cosh Q + \cos\phi \sinh Q & \sin\phi \sinh Q \\ \sin\phi \sinh Q & \cosh Q - \cos\phi \sinh Q \end{pmatrix}$$



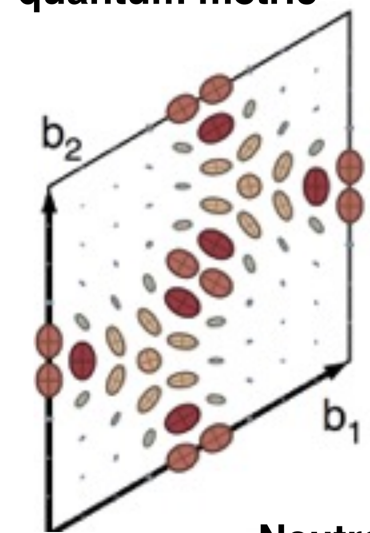
Can excite higher-spin modes by switching on a spin-4 perturbation

Liu, **AG**, Papic (2018)

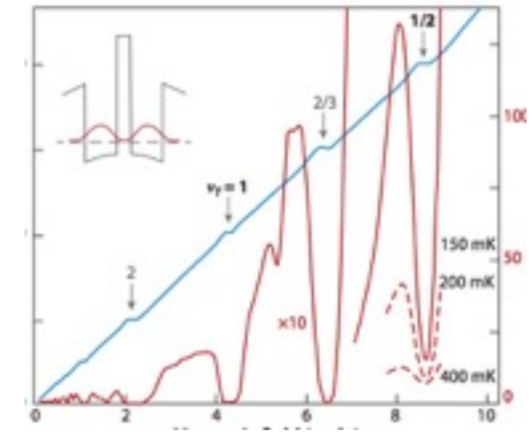
OPEN PROBLEMS

- ★ Higher spin degrees of freedom and anisotropy
- ★ Bimetric theory of Fractional Chern Insulators
- ★ Multi-layer Fractional Quantum Hall states
- ★ Neutral fermionic collective mode in 5/2 state
- ★ Higher spin formulation of CFL theory
- ★ Geometric theory of Quantum Hall liquid crystals
- ★ Relation to geometric structure of Fracton order

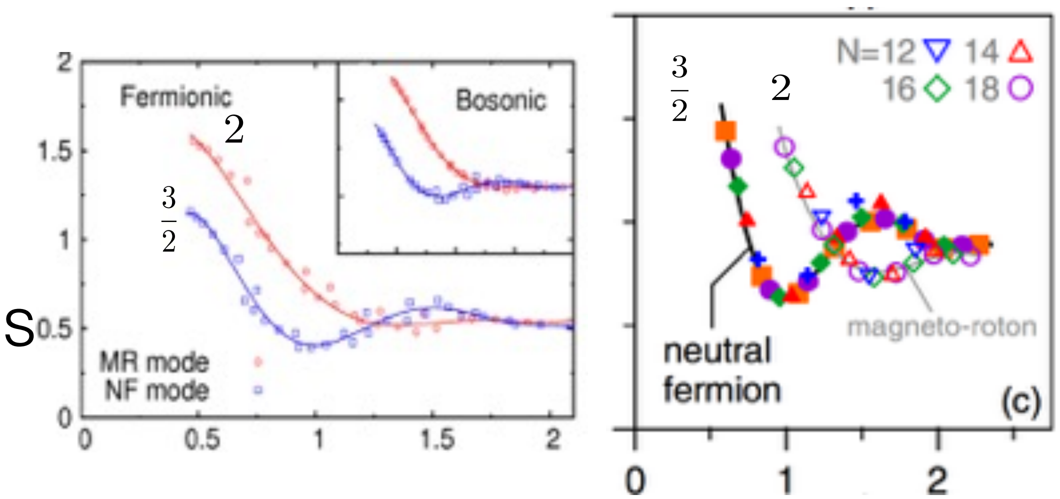
FCI
``quantum metric''



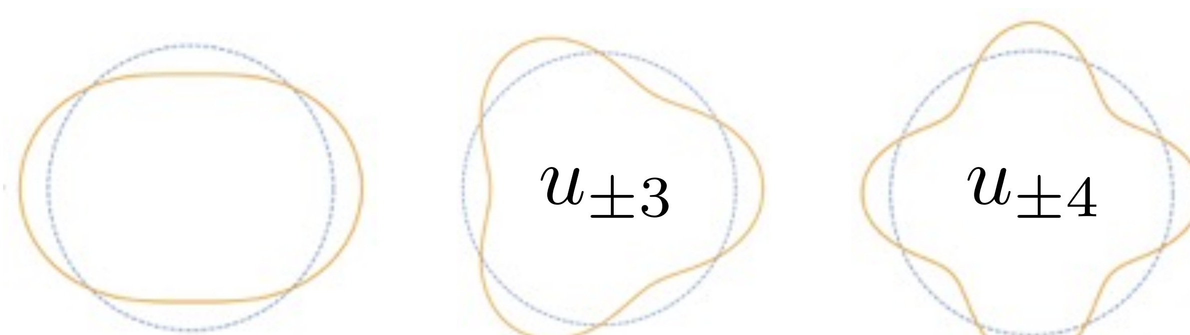
Bi-layer FQH



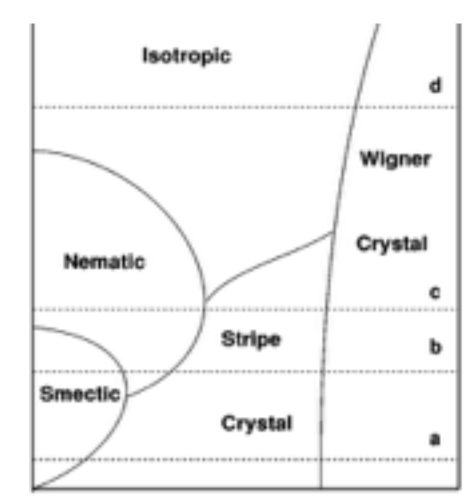
Neutral Fermion in 5/2 state



Higher spin modes



FQH liquid crystal



WHAT ELSE CAN BIMETRIC THEORY DO ?

- ★ *Projected* static structure factor up to $|\mathbf{k}|^6$
- ★ Dispersion relation of the GMP mode up to $|\mathbf{k}|^2$
- ★ *Absence* of the GMP mode and nematic transition in IQH
- ★ Manifest Particle-Hole duality
- ★ Girvin-MacDonald-Platzman algebra holds up to $|\mathbf{k}|^4$
- ★ DC Hall conductivity to $|\mathbf{k}|^2$
- ★ Hall viscosity to $|\mathbf{k}|^2$
- ★ Hints at rich structure of the full W_∞ theory

AG, Son 2017

GEOMETRIC COMPOSITE FERMI LIQUID THEORY

COMPOSITE FERMI LIQUID

States at filling $\nu = \frac{N}{2N+1}$ $\approx$ IQH of composite fermions at $\nu_{\text{eff}} = N$

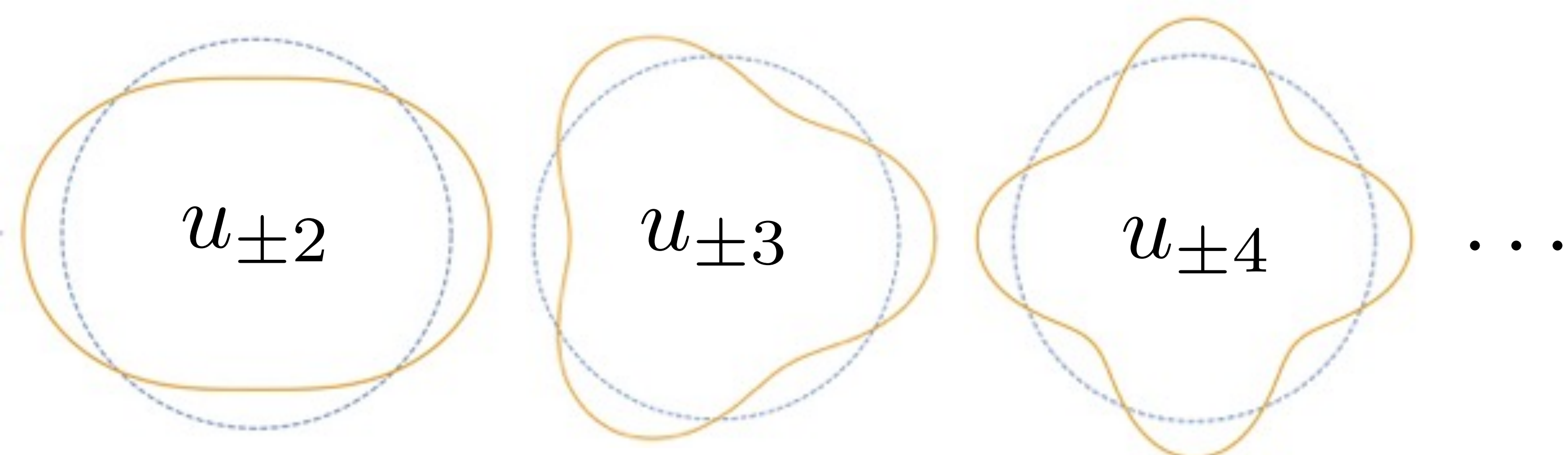
Can be treated via Fermi liquid theory when N is large

Semi-classically the d.o.f. are multipolar distortions of the Fermi surface

Dilation

Translation

non-dynamical



$\cdots$ dynamical

Shear

``Higher spin" area preserving deformations

Golkar Nguyen Roberts Son 2016

COMPOSITE FERMI LIQUID IN SMA

Hamiltonian

$$H = \frac{v_F k_F}{4\pi} \sum_n \int d^2\mathbf{x}(1 + F_n) u_n(\mathbf{x}) u_{-n}(\mathbf{x}),$$

Phenomenological ``Landau parameters''

CCR

$$[u_n(\mathbf{x}), u_m(\mathbf{x}')] = \frac{2\pi}{k_F^2}\left(n\bar{b}\delta_{n+m,0} - ik_F\delta_{n+m,1}\partial_{\bar{z}} - ik_F\delta_{n+m,-1}\partial_z\right)\delta(\mathbf{x}-\mathbf{x}')$$

All modes are gapped at $\Delta_n = n(1+F_n)\omega_c$

The limit $\Delta_2 \ll \Delta_n$ for all $n \geq 3$ is the SMA

Only dynamics of shear distortions $u_{\pm 2}$ remains

$$[u_2(\mathbf{x}), u_{-2}(\mathbf{x}')] = \frac{4\pi}{2N+1}\delta(\mathbf{x}-\mathbf{x}')$$

Same as linearized bimetric

Nguyen, AG, Son In Progress

COMPOSITE FERMI LIQUID IN SMA II

Effective Lagrangian for the quadrupolar (spin-2) mode

$$\mathcal{L}_{\rm SMA} = -\frac{i}{2}\frac{2N+1}{2\pi}u_2\dot{u}_{-2} + \frac{i}{2}\frac{N^2(2N+3)\ell^2}{12\pi}u_2\Delta\dot{u}_{-2} - \frac{c_0(2N+1)\omega_c}{2\pi}u_2u_{-2} + \frac{c_2(2N+1)\omega_c\ell^2}{2\pi}u_2\Delta u_{-2}$$

coincides with the linearized bimetric theory

$$\mathcal{L}_{\rm bm} = \frac{2N+1}{16\pi}Ad\hat{\omega} - \frac{N^2(2N+3)}{96\pi}\hat{\omega}d\hat{\omega} - \frac{\tilde{m}}{2}\left[\hat{g}_{ij}g^{ij} - \gamma\right]^2 - \frac{\alpha}{4}\left[\Gamma - \hat{\Gamma}\right]^2$$

Bimetric theory prescribes coupling of the CFL to curved space

<u>Conjecture:</u>

Bimetric theory is the geometric non-linear completion of the CFL <u>in the SMA</u>.

What about beyond SMA ?

GIRVIN - MACDONALD - PLATZMAN MODE I

Warning: not standard presentation

The LLL generators of W_∞ are $\mathcal{L}_{n,m} = \sum_{i=1}^{N_{\text{el}}} z_i^{n+1} \partial_{z_i}^{m+1}$

LLL Rotation

LLL Shears, spin-2 operators

Operators $\{\mathcal{L}_{0,0}, \mathcal{L}_{1,-1}, \mathcal{L}_{-1,1}\}$ form $\mathfrak{sl}(2,\mathbb{R})$ algebra

The projected density operator is expanded in $\mathcal{L}_{n,m}$

$$\bar{\rho}(\mathbf{k}) = e^{-\frac{|k|^2}{2}} \sum_{m,n} c_{nm} \bar{k}^n k^m \mathcal{L}_{n-1,m-1}$$

$\mathcal{L}_{n,m}$ create *intra*-LL state at momentum $\mathbf{k}$

GIRVIN - MACDONALD - PLATZMAN MODE II

At long wave-lengths the GMP mode is

$$\bar{\rho}(\mathbf{k})|0\rangle = \left[\frac{\mathrm{k}^2}{8}\mathcal{L}_{-1,1} + \frac{\bar{\mathrm{k}}^2}{8}\mathcal{L}_{1,-1} + \ldots\right]|0\rangle$$

The GMP state $\bar{\rho}(\mathbf{k})|0\rangle$ is a shear distortion at small $\mathbf{k}$

For IQH $\bar{H} = 0$ ⟶ $\bar{\rho}(\mathbf{k})|0\rangle$ is a 0 energy state

Consider two-body Hamiltonian $\bar{H} = \sum_{\mathbf{k}} V(\mathbf{k})\bar{\rho}(-\mathbf{k})\bar{\rho}(\mathbf{k})$

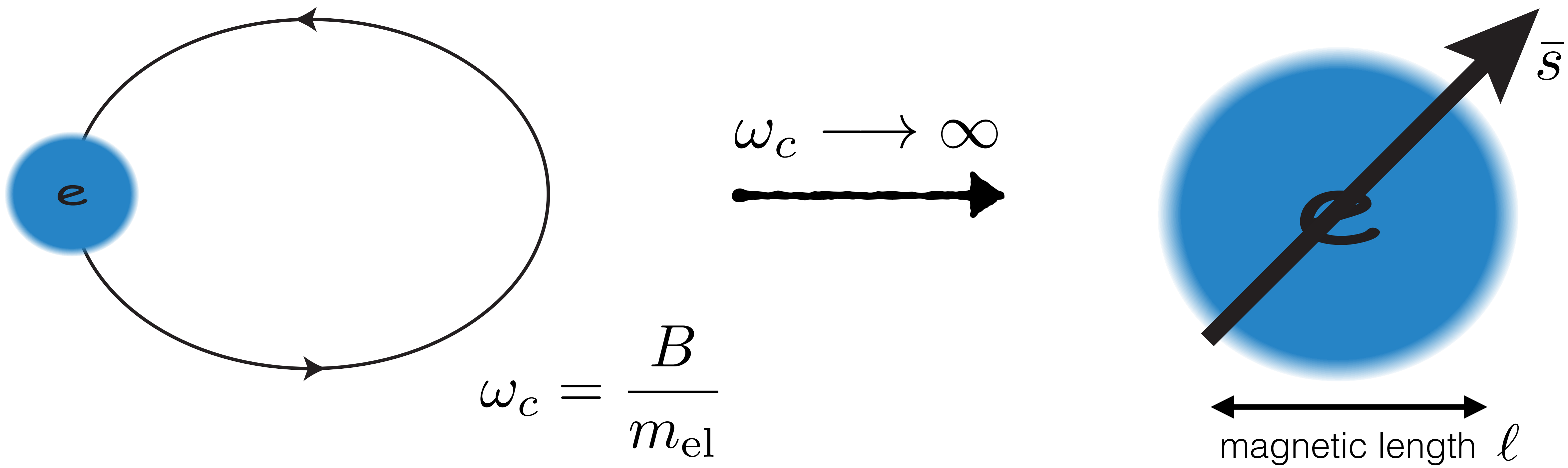
Since $[H, \bar{\rho}(\mathbf{k})] \neq 0$ the shear distortion costs energy

At small $\mathbf{k}$ GMP mode is a gapped, propagating, shear distortion of the FQH fluid

ORBITAL SPIN

In the remainder of the talk I will use the term ``orbital spin''.

In magnetic field electrons quickly move in cyclotron orbits



We consider the limit $m_{\mathrm{el}} \longrightarrow 0$

``Orbital spin'' describes the coupling of the low energy physics to spatial geometry

Read 2009

Wen Zee 1992

WHAT ELSE CAN BIMETRIC THEORY DO ?

Schematic Lagrangian up to three derivatives

$$\mathcal{L}_{\rm bm} = \frac{\nu\varsigma}{2\pi} A d\hat{\omega} - \frac{\hat{c}}{4\pi}\hat{\omega} d\hat{\omega} - \frac{\nu\varsigma}{4\pi}\hat{\nabla}_i E_i B - \frac{\hat{c}\ell^2}{8\pi}\hat{\nabla}_i E_i \hat{R} - \frac{\tilde{m}}{2}\left(\frac{1}{2}\hat{g}_{ij}g^{ij} - \gamma\right)^2 - \frac{\alpha}{4}\left|\Gamma - \hat{\Gamma}\right|^2$$

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