Non-Abelian spin-singlet states from coupled wires

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Fractional quantum Hall state

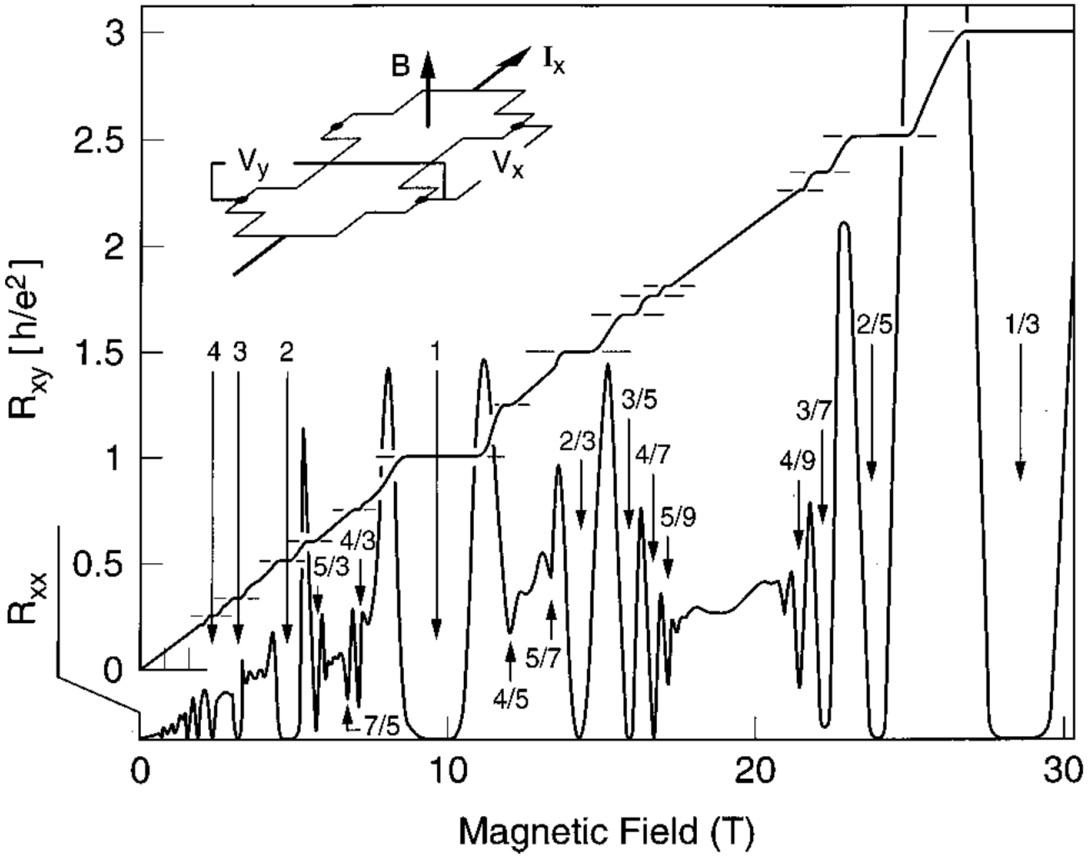
Topological order of strongly interacting particles

--- Quantized Hall conductance

--- Fractional charge & statistics

--- Topological degeneracy

--- Gapless chiral edge states



Eisenstein & Stormer (1990)

Halperin state

Trial wave function for single-component (fully polarized) electron

→ Laughlin $\nu = 1/m$ state: Laughlin (1983)

$$\Psi_m(z_1, \cdots, z_{N_e}) = \prod_{j<k} (z_j - z_k)^m e^{-\sum_{k=1}^{N_e} |z_j|^2/4l_0^2}$$

--- Hidden SU(2) symmetry for $m = 2$

Ansatz for unpolarized electron

→ Halperin mmn state at $\nu = 2/(m+n)$: Halperin (1983)

$$\Psi_{mmn}(\{z_i^\uparrow, z_i^\downarrow\}) = \prod_{j<k} (z_j^\uparrow - z_k^\uparrow)^m (z_j^\downarrow - z_k^\downarrow)^m \prod_{j,k} (z_j^\uparrow - z_k^\downarrow) e^{-\sum_k (|z_k^\uparrow|^2 + |z_k^\downarrow|^2)/4l_0^2}$$

--- Singlet under SU(2) for $n = m - 1$

--- Hidden SU(3) symmetry for $(m, n) = (2, 1)$

Hidden symmetry of Halperin state

Generalized Halperin state for $(N-1)$-component particles:

$$\Psi_{\mathbf{K}}(\{z_i^\sigma\}) = \prod_{\sigma}\prod_{j<k}(z_j^\sigma - z_k^\sigma)^{K_{\sigma\sigma}} \prod_{\sigma<\sigma'}\prod_{j,k}(z_j^\sigma - z_k^{\sigma'})^{K_{\sigma\sigma'}} e^{-\sum_{\sigma,k}|z_k^\sigma|^2/4l_0^2}$$

K: $(N-1)$-dim integer matrix appearing also in

--- Chern-Simons theory $\mathcal{L} = -\frac{i}{4\pi}K_{IJ}\epsilon_{\mu\nu\lambda}\alpha_\mu^I\partial_\nu\alpha_\lambda^J - \frac{i}{2\pi}t_I\epsilon_{\mu\nu\lambda}A_\mu\partial_\nu\alpha_\lambda^I$

--- Edge theory $[\partial_x\tilde{\phi}_I(x), \tilde{\phi}_J(y)] = 2\pi i K_{IJ}\delta(x-y)$

$$\mathbf{K} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2 & & & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 1 & & & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{pmatrix}.$$

--- Filling $\nu = (N-1)/N$

--- Singlet under SU(N-1)

--- Transformed into Cartan matrix of SU(N)

→ Underlaying CFT is level-1 SU(N) WZW.

Cf. Regnault, Goerbig, & Jolicoeur (2008)

FQHS-CFT correspondence

Moore & Read (1992)

Trial wave function of FQHS

$$\Psi(z_1, z_2, \cdots, z_N)$$

$z_i = x + iy$: Spatial coordinate for i-th electron

Correlation function of CFT

$$\langle V_e(z_1) V_e(z_2) \cdots V_e(z_N) V_{-Ne} \rangle$$

$V_e(z)$: Current of CFT, V_{-Ne}: Neutralization factor

Fractional quantum Hall state (FQHS)	Conformal field theory (CFT)
Electron operators	Simple currents $V_e(z)$
Quasiparticle operators	Primary fields
GS degeneracy on torus	Number of primary fields
Statistics of quasiparticles	Modular T & S matrices
Coefficient of thermal Hall conductance	Central charge

→ The same CFT also describes edge.

Non-Abelian spin-singlet state

Ardonne & Schoutens (1999);
Ardonne, Read, Rezayi, & Schoutens (2001)

Divide bosons into k groups, forming the Halperin state in each group, and symmetrize over all possible partitions.

$$\tilde{\Psi}_{SU(N)_k}(\{z_i^\sigma\}) = \mathcal{S}_{\text{groups}} \prod_{\text{groups}} \tilde{\Psi}_{\mathbf{K}}. \qquad \nu = \frac{k(N-1)}{N}$$

Cappelli, Georgiev, & Todorov (2001);
Sterdyniak, Repellin, Bernevig, & Regnault (2013)

--- Underlying CFT: $[SU(N)_1]^k \to SU(N)_k$

--- Neutral sector is non-Abelian: Gepner parafermion $SU(N)_k/[U(1)]^{N-1}$ Gepner (1987)

--- $N=2$: Bosonic Read-Rezayi, $N=3$: Bosonic NASS at $\nu = 2k/3$

Read & Rezayi (1999);
Ardonne & Schoutens (1999)

Ex) Fusion rule of $SU(3)_2$ parafermion

$\{1\}$	$\{\psi_1\}$	$\{\psi_2\}$	$\{\psi_{12}\}$	$\{\rho\}$	$\{\sigma_3\}$	$\{\sigma_1\}$	$\{\sigma_2\}$
$\{\psi_1\}$	$\{1\}$	$\{\psi_{12}\}$	$\{\psi_2\}$	$\{\sigma_3\}$	$\{\rho\}$	$\{\sigma_2\}$	$\{\sigma_1\}$
$\{\psi_2\}$	$\{\psi_{12}\}$	$\{1\}$	$\{\psi_1\}$	$\{\sigma_1\}$	$\{\sigma_2\}$	$\{\rho\}$	$\{\sigma_3\}$
$\{\psi_{12}\}$	$\{\psi_2\}$	$\{\psi_1\}$	$\{1\}$	$\{\sigma_2\}$	$\{\sigma_1\}$	$\{\sigma_3\}$	$\{\rho\}$
$\{\rho\}$	$\{\sigma_3\}$	$\{\sigma_1\}$	$\{\sigma_2\}$	$\{1, \rho\}$	$\{\psi_1, \sigma_3\}$	$\{\psi_2, \sigma_1\}$	$\{\psi_{12}, \sigma_2\}$
$\{\sigma_3\}$	$\{\rho\}$	$\{\sigma_2\}$	$\{\sigma_1\}$	$\{\psi_1, \sigma_3\}$	$\{1, \rho\}$	$\{\psi_{12}, \sigma_2\}$	$\{\psi_2, \sigma_1\}$
$\{\sigma_1\}$	$\{\sigma_2\}$	$\{\rho\}$	$\{\sigma_3\}$	$\{\psi_2, \sigma_1\}$	$\{\psi_{12}, \sigma_2\}$	$\{1, \rho\}$	$\{\psi_1, \sigma_3\}$
$\{\sigma_2\}$	$\{\sigma_1\}$	$\{\sigma_3\}$	$\{\rho\}$	$\{\psi_{12}, \sigma_2\}$	$\{\psi_2, \sigma_1\}$	$\{\psi_1, \sigma_3\}$	$\{1, \rho\}$

--- Potential realization in multilayer, cold atom, fractional Chern insulator,,,

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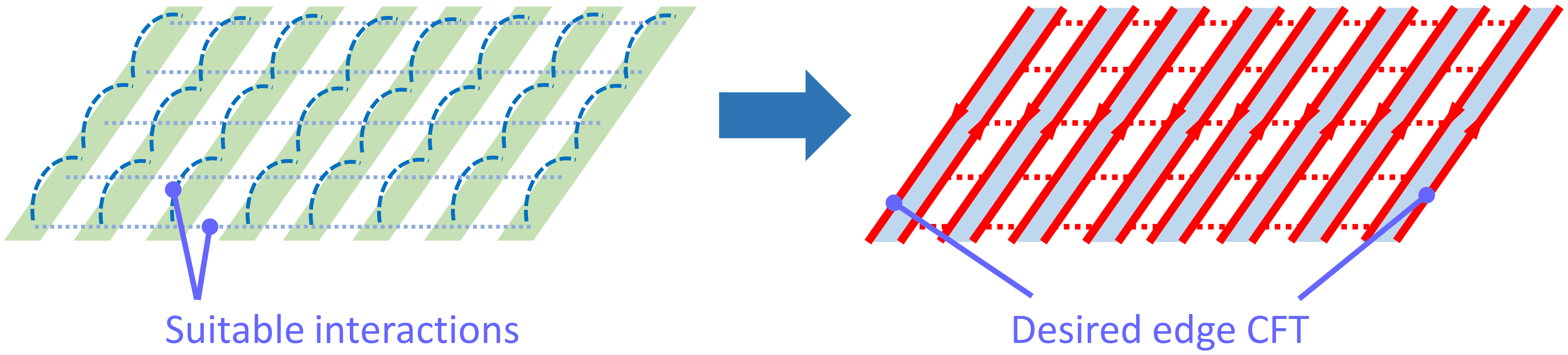
Coupled-wire construction

Kane, Mukhopadhyay, & Lubensky (2002); Teo & Kane (2014)

Luttinger liquid = free boson theory



--- We can obtain microscopic Hamiltonian.

--- We can study edge states, quasiparticles, GS degeneracy,,,

Sagi, Oreg, Stern, & Halperin (2015)

--- Possible connection with lattice systems.

Gorohovsky, Pereira, & Sela (2015); YF, He, Bhattacharjee, & Pollmann (2016); Lecheminant & Tsvelik (2017); Chen, Mudry, Chamon, & Tsvelik (2017)

Strategy for NASS

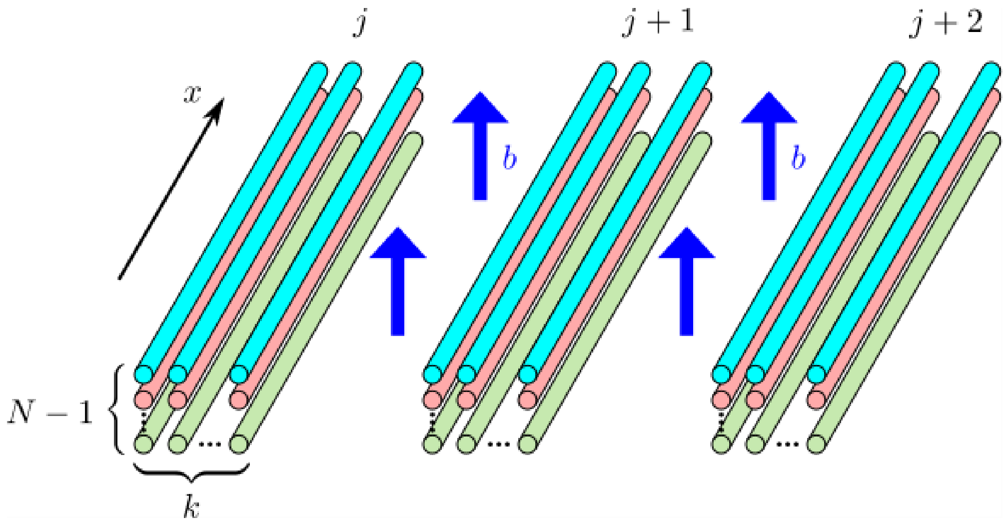
1) Construct Abelian state from (N-1)-component bosonic wires.

CFT: $SU(N)_1$

$$\mathbf{K} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2 & & & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 1 & & & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{pmatrix}.$$

2) Stack k copies of Abelian state and find appropriate interactions.

CFT: $[SU(N)_1]^k \sim SU(N)_k \times \frac{[SU(N)_1]^k}{SU(N)_k} \sim SU(N)_k \times \frac{SU(k)_N}{[U(1)]^{k-1}}$



Strategy for NASS

1) Construct Abelian state from (N-1)-component bosonic wires.

$$\mathbf{K} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2 & & & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 1 & & & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{pmatrix}.$$

CFT: $SU(N)_1$

2) Stack k copies of Abelian state and find appropriate interactions.

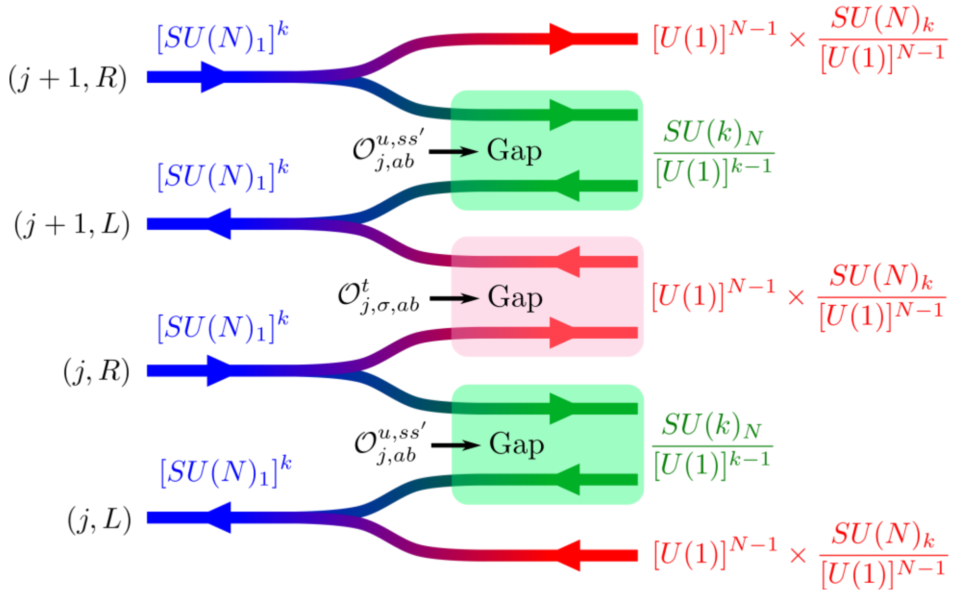
$$\text{CFT: } [SU(N)_1]^k \sim SU(N)_k \times \frac{[SU(N)_1]^k}{SU(N)_k} \sim SU(N)_k \times \frac{SU(k)_N}{[U(1)]^{k-1}}$$

Interwire tunnelings between k copies

→ Gap in $SU(N)_k$

Intrawire interactions between k copies.

→ Gap in $\frac{SU(k)_N}{[U(1)]^{k-1}}$



Abelian state

Sliding Luttinger liquid:

$$\mathcal{H}_{\rm SLL} = \mathcal{H}_0 + \mathcal{H}_{\rm forward}. \qquad \mathcal{H}_0 = \sum_{j=1}^{N_w}\sum_{\sigma=1}^{N-1} \frac{v_F}{2\pi}\int dx \left[\frac{1}{g}(\partial_x\theta_{j,\sigma})^2 + g(\partial_x\varphi_{j,\sigma})^2\right],$$

$$[\theta_{j,\sigma}(x), \varphi_{j',\sigma'}(x')] = i\pi\delta_{jj'}\delta_{\sigma\sigma'}\Theta(x-x'),$$

Interwire tunneling at $\nu = (N-1)/N$ with charge & momentum conservation:

$$\begin{aligned}\mathcal{O}^t_{j,\sigma} &= \exp i\left[\varphi_{j,\sigma} - \varphi_{j+1,\sigma} + \sum_{\sigma'=1}^{N-1}\left(M_{\sigma\sigma'}\theta_{j,\sigma'} + M^T_{\sigma\sigma'}\theta_{j+1,\sigma'}\right)\right], \\ &= \exp i\left(\tilde{\phi}^R_{j,\sigma} - \tilde{\phi}^L_{j+1,\sigma}\right)\end{aligned}$$

$$\mathbf{M} = \begin{pmatrix} 2 & 2 & \cdots & 2 & 2 \\ 0 & 2 & & 2 & 2 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 2 & 2 \\ 0 & 0 & \cdots & 0 & 2 \end{pmatrix},$$

→ Edge theory with desired commutation relation

$$[\partial_x\tilde{\phi}^p_{j,\sigma}(x), \tilde{\phi}^{p'}_{j',\sigma'}(x')] = 2i\pi p\delta_{pp'}\delta_{jj'}K_{\sigma\sigma'}\delta(x-x')$$

$$\mathbf{K} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2 & & & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 1 & & & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{pmatrix}.$$

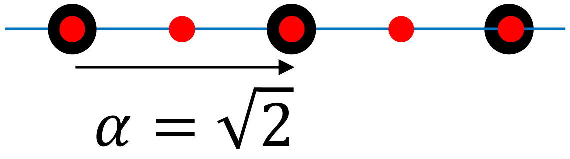
Abelian state

K matrix = Gram matrix specifying an $(N-1)$-dim lattice   Read (1990)

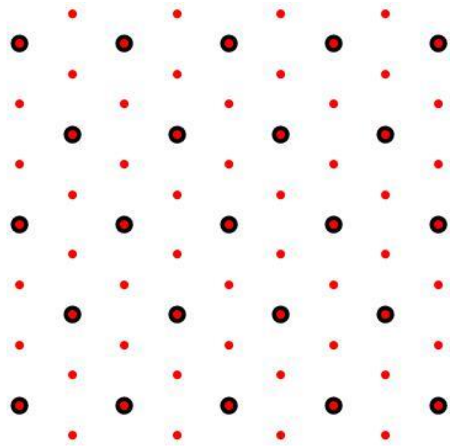
$$K_{\sigma\sigma'} = \boldsymbol{\alpha}_\sigma \cdot \boldsymbol{\alpha}_{\sigma'}$$

Root lattice of SU(N): $\mathbb{Z}\boldsymbol{\alpha}_1 + \cdots + \mathbb{Z}\boldsymbol{\alpha}_{N-1} \ni \boldsymbol{\alpha}$

$SU(2)$



$SU(3)$



If SLL Hamiltonian is fine tuned,

Basis diagonalizing K:

$$[\partial_x \tilde{\chi}^p_{j,l}(x), \tilde{\chi}^{p'}_{j',l'}(x')] = 2ip\pi\delta_{pp'}\delta_{jj'}\delta_{ll'}\delta(x-x').$$

$SU(N)_1$ current algebra:

$$H^l_j(x) = \partial_x \tilde{\chi}^R_{j,l}(x) \qquad E^{\pm\boldsymbol{\alpha}}_j(x) = \frac{\pm i}{x_c} e^{\pm i\boldsymbol{\alpha}\cdot\tilde{\boldsymbol{\chi}}^R_j(x)}$$

$$\mathcal{H}_{\mathrm{SLL}} = \frac{v}{4\pi(N+1)} \sum_{j=1}^{N_w} \int dx \left[\sum_{l=1}^{N-1} :H^l_j H^l_j: + \sum_{\boldsymbol{\alpha}\in\Delta_N} :E^{\boldsymbol{\alpha}}_j E^{-\boldsymbol{\alpha}}_j: + \sum_{l=1}^{N-1} :\bar{H}^l_j \bar{H}^l_j: + \sum_{\boldsymbol{\alpha}\in\Delta_N} :\bar{E}^{\boldsymbol{\alpha}}_j \bar{E}^{-\boldsymbol{\alpha}}_j: \right],$$

Abelian state

SLL Hamiltonian → $SU(N)_1$ WZW CFT

$$\mathcal{H}_{\mathrm{SLL}} = \frac{v}{4\pi(N+1)} \sum_{j=1}^{N_w} \int dx \left[\sum_{l=1}^{N-1} :H_j^l H_j^l: + \sum_{\boldsymbol{\alpha}\in\Delta_N} :E_j^{\boldsymbol{\alpha}} E_j^{-\boldsymbol{\alpha}}: + \sum_{l=1}^{N-1} :\bar{H}_j^l \bar{H}_j^l: + \sum_{\boldsymbol{\alpha}\in\Delta_N} :\bar{E}_j^{\boldsymbol{\alpha}} \bar{E}_j^{-\boldsymbol{\alpha}}: \right],$$

Interwire interaction → current-current perturbation

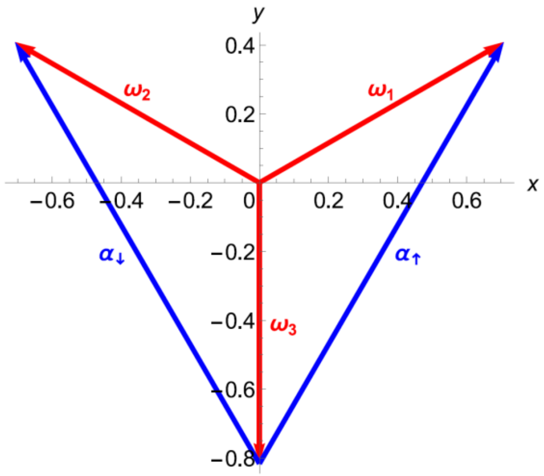
$$\mathcal{O}_{j,\sigma}^t(x) = x_c^2 E_j^{\boldsymbol{\alpha}_\sigma}(x) \bar{E}_{j+1}^{-\boldsymbol{\alpha}_\sigma}(x).$$

Quasiparticle operators → $SU(N)_1$ primaries

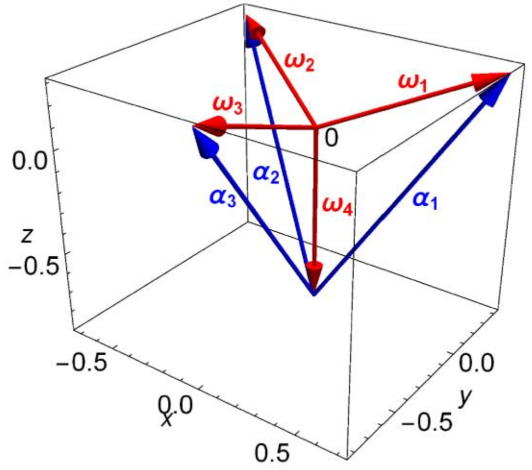
$$e^{i2\theta_{j,\sigma}(x)} = e^{i\boldsymbol{\omega}_\sigma \cdot \tilde{\boldsymbol{\chi}}_j^R(x) - i\boldsymbol{\omega}_\sigma \cdot \tilde{\boldsymbol{\chi}}_j^L(x)}$$

ω: Weight of SU(N)

SU(3)



SU(4)



--- SU(N) structure is manifest in Hamiltonian.

Non-Abelian spin-singlet state

Stack k copies of Abelian states. → $k(N-1)$ bosonic fields in each wire

$$\mathcal{H}_{\mathrm{SLL}} = \sum_{j=1}^{N_w}\sum_{a=1}^{k} \frac{v}{4\pi}\int dx\,\left[(\partial_x \tilde{\mathbf{\chi}}_{j,a}^{R})^2 + (\partial_x \tilde{\mathbf{\chi}}_{j,a}^{L})^2\right].$$

$$= \frac{v}{4\pi}\sum_{j=1}^{N_w}\int dx \Bigg[\underbrace{(\partial_x \tilde{\mathbf{X}}_j^{R})^2 + (\partial_x \tilde{\mathbf{X}}_j^{L})^2}_{N-1 \text{ charge modes}} + \sum_{\mu=1}^{k-1}\underbrace{\left\{(\partial_x \tilde{\mathbf{Y}}_j^{R,\mu})^2 + (\partial_x \tilde{\mathbf{Y}}_j^{L,\mu})^2\right\}}_{(N-1)(k-1) \text{ neutral modes}}\Bigg],$$

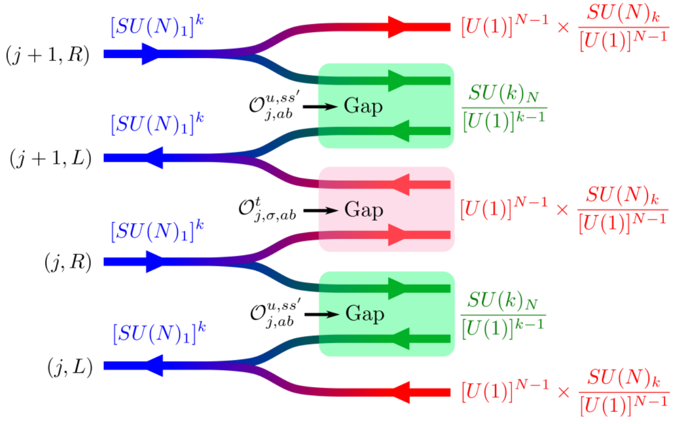
$N-1$ charge modes $\qquad (N-1)(k-1)$ neutral modes

→ $[U(1)]^{N-1}$ boson CFT $\qquad$ → $\dfrac{SU(N)_k}{[U(1)]^{N-1}} \times \dfrac{SU(k)_N}{[U(1)]^{k-1}}$ parafermion CFTs

We want to gap out

$SU(N)_k \sim [U(1)]^{N-1} \times \dfrac{SU(N)_k}{[U(1)]^{N-1}}$ by *interwire* interactions

$\dfrac{SU(k)_N}{[U(1)]^{k-1}}$ by *intrawire* interactions



Non-Abelian spin-singlet state

Interwire interactions: electron tunnelings between k copies

$$\sum_{a=1}^{k}\sum_{b=1}^{k} E_{j,a}^{\boldsymbol{\alpha}_\sigma}(x)\bar{E}_{j+1,b}^{-\boldsymbol{\alpha}_\sigma}(x) = \mathcal{E}_j^{\boldsymbol{\alpha}_\sigma}\bar{\mathcal{E}}_{j+1}^{-\boldsymbol{\alpha}_\sigma}(x)$$

← $SU(N)_k$ currents

$$\sim e^{i\boldsymbol{\alpha}_\sigma\cdot(\tilde{\boldsymbol{X}}_j^R(x)-\tilde{\boldsymbol{X}}_{j+1}^L(x))/\sqrt{k}}\Psi_j^{\boldsymbol{\alpha}_\sigma,1}(x)\bar{\Psi}_{j+1}^{-\boldsymbol{\alpha}_\sigma,1}(x)$$

→ Gapping $SU(N)_k \sim [U(1)]^{N-1} \times \frac{SU(N)_k}{[U(1)]^{N-1}}$

$\frac{SU(N)_k}{[U(1)]^{N-1}}$ parafermion fields

Vertex representation for parafermion: Dunne, Halliday, & Suranyi (1989)

Intrawire interactions: quasiparticle interactions between k copies

$$\sum_{a<b}\sum_{s,s'=1}^{N} u_{ab}e^{i\boldsymbol{\omega}_s\cdot(\tilde{\boldsymbol{\chi}}_{j,a}^R(x)-\tilde{\boldsymbol{\chi}}_{j,b}^R(x))-i\boldsymbol{\omega}_{s'}\cdot(\tilde{\boldsymbol{\chi}}_{j,a}^L(x)-\tilde{\boldsymbol{\chi}}_{j,b}^L(x))} = \sum_{\boldsymbol{A}\in\Delta_k^+}\frac{u_{\boldsymbol{A}}}{2}\Xi_j^{\boldsymbol{A}}(x)\bar{\Xi}_j^{\boldsymbol{A}\dagger}(x)$$

A: root of SU(k)

$\frac{SU(k)_N}{[U(1)]^{k-1}}$ parafermion fields

→ Gapping $\frac{SU(k)_N}{[U(1)]^{k-1}}$ parafermion CFT???

If gapped, how can we read off non-Abelian quasiparticles???

Non-Abelian spin-singlet state

Intrawire interaction is integrable for k=2, but no solution is known for general *N* & *k*.

Fateev (1991); Fateev & Zamolodchikov (1991)

Looking at neutral sector of a single wire:

$$
\begin{aligned}
\mathcal{H}_{\text{neutral}} &= \mathcal{H}_{SU(N)_k/[U(1)]^{N-1}} + \mathcal{H}_{SU(k)_N/[U(1)]^{k-1}} + \int dx \sum_{\boldsymbol{A}\in\Delta_k^+} \frac{u_{\boldsymbol{A}}}{2} \Xi^{\boldsymbol{A}}(x)\bar{\Xi}^{\boldsymbol{A}\dagger}(x) \\
&= \frac{v}{4\pi}\int dx \sum_{\mu=1}^{k-1}\left[(\partial_x \tilde{\boldsymbol{Y}}^{R,\mu})^2 + (\partial_x \tilde{\boldsymbol{Y}}^{L,\mu})^2\right] + \int dx \sum_{s,s'=1}^{N}\sum_{\boldsymbol{A}\in\Delta_k^+} u_{\boldsymbol{A}} \cos(\boldsymbol{\omega}_s \boldsymbol{A}\cdot\tilde{\boldsymbol{Y}}^R - \boldsymbol{\omega}_{s'}\boldsymbol{A}\cdot\tilde{\boldsymbol{Y}}^L) \\
&= \frac{v}{2\pi}\int dx \sum_{\mu=1}^{k-1}\left[(\partial_x \tilde{\boldsymbol{\Phi}}^{\mu})^2 + (\partial_x \tilde{\boldsymbol{\Theta}}^{\mu})^2\right] + \int dx \sum_{s=1}^{N}\sum_{\boldsymbol{A}\in\Delta_k^+} u_{\boldsymbol{A}} \cos(2\boldsymbol{\omega}_s \boldsymbol{A}\cdot\tilde{\boldsymbol{\Theta}}) + \int dx \sum_{s\neq s'}\sum_{\boldsymbol{A}\in\Delta_k^+} u_{\boldsymbol{A}} \cos\Big[(\boldsymbol{\omega}_s - \boldsymbol{\omega}_{s'})\boldsymbol{A}\cdot\tilde{\boldsymbol{\Phi}} + (\boldsymbol{\omega}_s + \boldsymbol{\omega}_{s'})\boldsymbol{A}\cdot\tilde{\boldsymbol{\Theta}}\Big],
\end{aligned}
$$

Non-Abelian spin-singlet state

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Fateev (1991); Fateev & Zamolodchikov (1991)

Looking at neutral sector of a single wire:

$$\mathcal{H}_{\mathrm{neutral}} = \frac{v}{2\pi}\int dx \sum_{\mu=1}^{k-1}\left[(\partial_x \tilde{\boldsymbol{\Phi}}^{\mu})^2 + (\partial_x \tilde{\boldsymbol{\Theta}}^{\mu})^2\right] + \int dx \sum_{s=1}^{N}\sum_{\boldsymbol{A}\in\Delta_k^+} u_{\boldsymbol{A}} \cos(2\boldsymbol{\omega}_s \boldsymbol{A}\cdot\tilde{\boldsymbol{\Theta}}) + \int dx \sum_{s\neq s'}\sum_{\boldsymbol{A}\in\Delta_k^+} u_{\boldsymbol{A}} \cos\Big[(\boldsymbol{\omega}_s - \boldsymbol{\omega}_{s'})\boldsymbol{A}\cdot\tilde{\boldsymbol{\Phi}} + (\boldsymbol{\omega}_s + \boldsymbol{\omega}_{s'})\boldsymbol{A}\cdot\tilde{\boldsymbol{\Theta}}\Big],$$

→ $(Z_k)^{N-1}$ symmetry breaking [$(Z_k)^{N-1}$ is symmetry of $\frac{SU(N)_k}{[U(1)]^{N-1}}$ parafermion.]

--- Backscattering operator $e^{i2\theta_{j,\sigma,a}(x)}$ creates excitations with charge 1/N.

--- Neutral sector of $e^{i2\theta_{j,\sigma,a}(x)}$ detects $(Z_k)^{N-1}$ symmetry breaking.

→ Order parameter field (twist field)

Hamiltonian is self *N*-al sine-Gordon model.

→ Property of $\frac{SU(N)_k}{[U(1)]^{N-1}}$ parafermion criticality.

Stat-mech model:
Bazhanov, Kashaev, Mangazeev, & Stroganov (1991);
Kashaev, Mangazeev, & Nakanishi (1991)

←→ $\frac{SU(k)_N}{[U(1)]^{k-1}}$ is gapped.

Non-Abelian spin-singlet state

$$\sum_{a=1}^{k}\sum_{b=1}^{k} E^{\boldsymbol{\alpha}_\sigma}_{j,a}(x)\bar{E}^{-\boldsymbol{\alpha}_\sigma}_{j+1,b}(x) = \mathcal{E}^{\boldsymbol{\alpha}_\sigma}_j \bar{\mathcal{E}}^{-\boldsymbol{\alpha}_\sigma}_{j+1}(x)$$
$$\sim e^{i\boldsymbol{\alpha}_\sigma\cdot(\tilde{\boldsymbol{X}}^R_j(x)-\tilde{\boldsymbol{X}}^L_{j+1}(x))/\sqrt{k}}\Psi^{\boldsymbol{\alpha}_\sigma,1}_j(x)\bar{\Psi}^{-\boldsymbol{\alpha}_\sigma,1}_{j+1}(x)$$

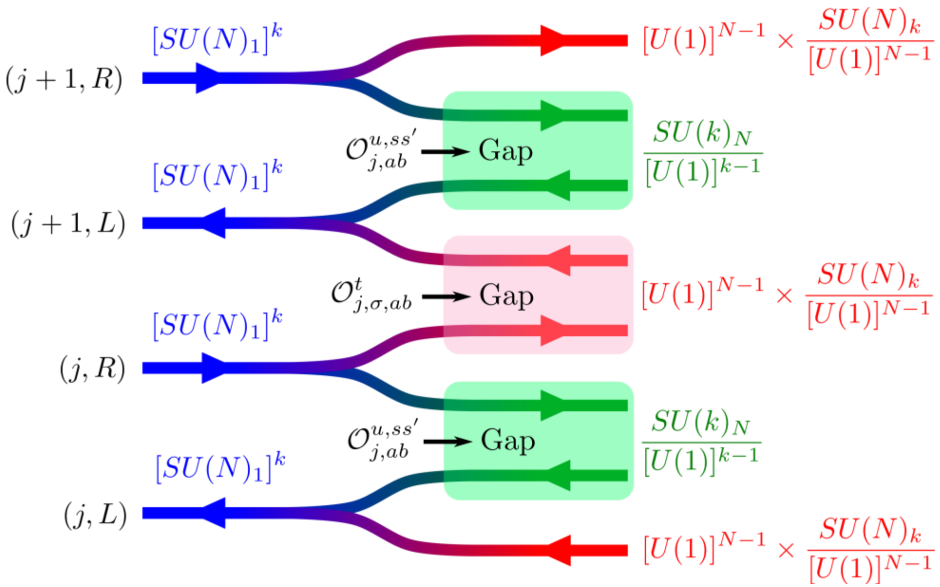
→ Gapping $SU(N)_k \sim [U(1)]^{N-1} \times \frac{SU(N)_k}{[U(1)]^{N-1}}$

$$\sum_{a<b}\sum_{s,s'=1}^{N} u_{ab} e^{i\boldsymbol{\omega}_s\cdot(\tilde{\boldsymbol{\chi}}^R_{j,a}(x)-\tilde{\boldsymbol{\chi}}^R_{j,b}(x)) - i\boldsymbol{\omega}_{s'}\cdot(\tilde{\boldsymbol{\chi}}^L_{j,a}(x)-\tilde{\boldsymbol{\chi}}^L_{j,b}(x))} = \sum_{\boldsymbol{A}\in\Delta_k^+}\frac{u_{\boldsymbol{A}}}{2}\Xi^{\boldsymbol{A}}_j(x)\bar{\Xi}^{\boldsymbol{A}\dagger}_j(x)$$

→ Gapping $\frac{SU(k)_N}{[U(1)]^{k-1}}$ parafermion CFT (conjecture)

Charge-1/N quasiparticles have twist field in neutral sector.

→ **Non-Abelian spin-singlet state**



General NASS

1) Construct $k(N-1)$-layer Abelian state at $\nu = \frac{k(N-1)}{N+k(N-1)m}$.

$$
\mathbf{K} = \begin{pmatrix} \mathbf{K}_{SU(N)} & & & \\ & \mathbf{K}_{SU(N)} & & \\ & & \ddots & \\ & & & \mathbf{K}_{SU(N)} \end{pmatrix} + m\mathbf{C}_{k(N-1)} \qquad \mathbf{K}_{SU(N)} = \begin{pmatrix} 2 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 2 & & & 1 \\ \vdots & \vdots & & \ddots & & \vdots \\ 1 & 1 & & & 2 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 2 \end{pmatrix}
$$

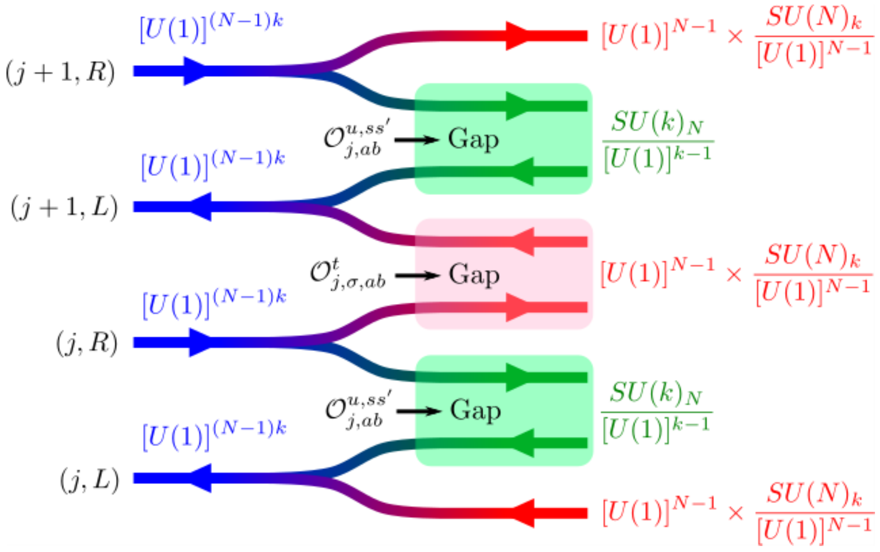
2) Find appropriate interactions: transition from Abelian to non-Abelian.

Interwire tunnelings between k copies

→ Gap in $[U(1)]^{N-1} \times \frac{SU(N)_k}{[U(1)]^{N-1}}$

Intrawire interactions between k copies.

→ Gap in $\frac{SU(k)_N}{[U(1)]^{k-1}}$



General NASS

NASS at $\nu = \frac{k(N-1)}{N+k(N-1)m}$: Same statistics in neutral sector, but charge is modified.

$$\mathbf{K} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\nu = \frac{4}{4m+3}$$

$$\mathbf{K} = \begin{pmatrix} m+2 & m+1 & m & m \\ m+1 & m+2 & m & m \\ m & m & m+2 & m+1 \\ m & m & m+1 & m+2 \end{pmatrix}$$

→

Ardonne-Schoutens:

--- Fractional charge $e/(4m+3)$

--- Unfractional spin ± 1

Ardonne & Schoutens (1999)

$$\mathbf{K}'_{SU(3)} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\nu = \frac{4}{4m+1}$$

$$\mathbf{K} = \begin{pmatrix} m+2 & m-1 & m & m \\ m-1 & m+2 & m & m \\ m & m & m+2 & m-1 \\ m & m & m-1 & m+2 \end{pmatrix}$$

→

Barkeshli-Wen:

--- Fractional charge $e/(4m+1)$

--- Fractional spin $\pm 1/3$

Barkeshli & Wen (2012)

$\mathbf{K}_{SU(3)} = \begin{pmatrix} -2 & -1 \\ -1 & -2 \end{pmatrix}$ or $\begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$ → Counterflow of parafermion

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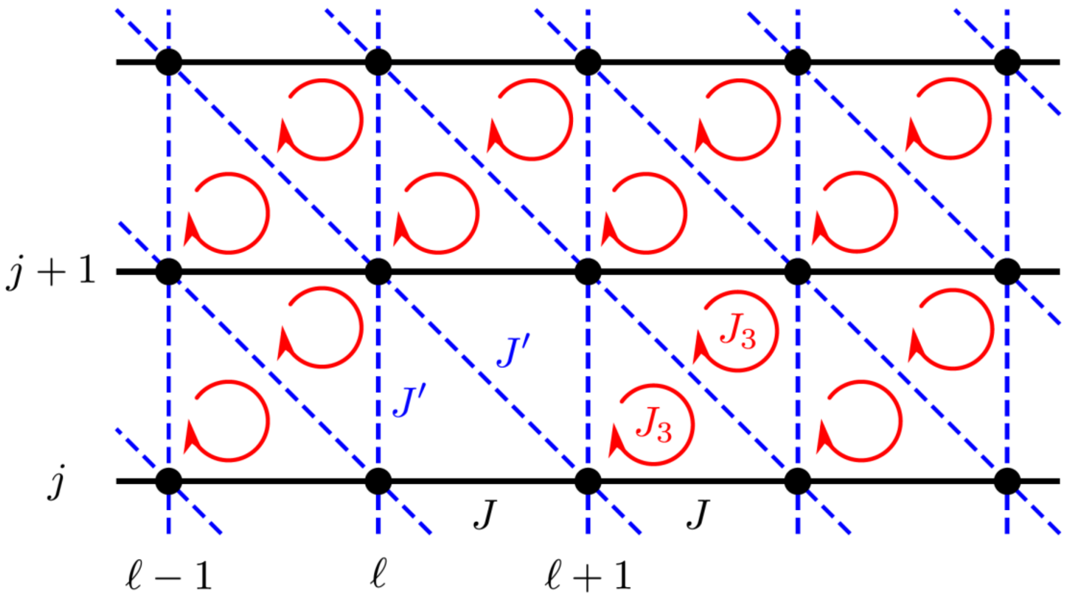
SU(N) chiral spin liquid

S=1/2: Gorohovsky, Pereira, & Sela (2015)



Coupled SU(N) Heisenberg chains:

$$\mathcal{H}_{\text{Heis}} = J \sum_{j,\ell} \sum_{a=1}^{N^2-1} T^a_{j,\ell} T^a_{j,\ell+1}$$

$$\mathcal{H}_1 = \sum_{j,\ell} \Bigg[J' \sum_{a=1}^{N^2-1} \left(T^a_{j,\ell} T^a_{j+1,\ell} + T^a_{j,\ell+1} T^a_{j+1,\ell} \right) + J_3 \sum_{a,b,c} f_{abc} \left(T^a_{j,\ell} T^b_{j+1,\ell} T^c_{j,\ell+1} + T^a_{j,\ell} T^b_{j+1,\ell-1} T^c_{j+1,\ell} \right) \Bigg]$$

Low energy

$$\mathcal{H}_0 = \frac{v}{4\pi(N+1)} \int dx \sum_j \left(:\mathbf{J}_j \cdot \mathbf{J}_j: + :\bar{\mathbf{J}}_j \cdot \bar{\mathbf{J}}_j: \right),$$

Tuning J_3 & J'

Long-range order (absent for N=2)

$$\mathcal{H}_1 \sim \int dx \sum_j \Big[2J' a_0 \left(\mathbf{J}_j \cdot \bar{\mathbf{J}}_{j+1} + \bar{\mathbf{J}}_j \cdot \mathbf{J}_{j+1} \right) + J' \left((1 + e^{\frac{2i\pi}{N}}) \mathbf{N}_j \cdot \mathbf{N}^\dagger_{j+1} + \text{H.c.} \right) + 4N J_3 a_0 \left(\mathbf{J}_j \cdot \bar{\mathbf{J}}_{j+1} - \bar{\mathbf{J}}_j \cdot \mathbf{J}_{j+1} \right) \Big] + \cdots$$

Chiral spin liquid

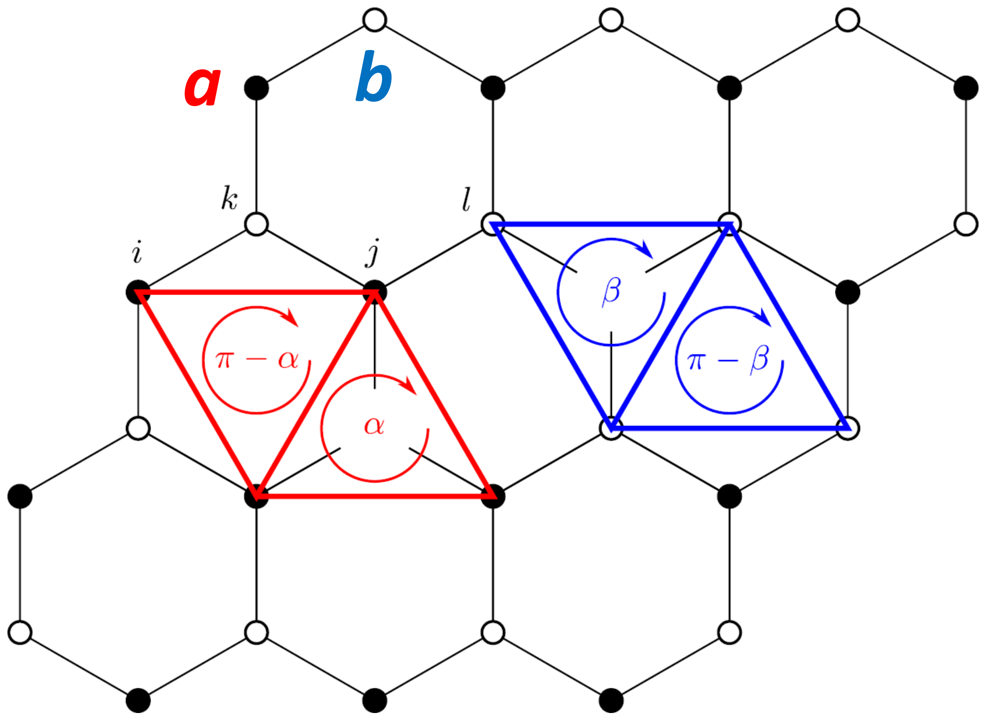
→ CSL with $SU(N)_1$ CFT with TRS breaking may be stabilized for large N.

Correlated hopping model

Hard-core bosons with correlated hopping:

He, Bhattacharjee, Moessner, Pollmann (2015);
YF, He, Bhattacharjee, & Pollmann (2016)

$$H = t \sum_{\langle ij;k \rangle} e^{iA_{ij}} a_i^\dagger a_j (2n_k^b - 1) + t \sum_{\langle kl;i \rangle} e^{iA_{kl}} b_k^\dagger b_l (2n_i^a - 1) + \mathrm{H.c.}$$



Mutual (reverse) flux attachment → Halperin 221 state at $\rho = 1/6$.

Summary & Outlook

Non-Abelian FQH states with $SU(N)_k$ parafermions are built out of parent Abelian FQH states with suitable interactions.

--- Analogue of 331 → Pfaffian, 113 → PH-Pfaffian

Relation with $(Z_k)^{N-1}$ stat-mech model and self-*N*-al sine-Gordon model.

YF & P. Lecheminant, Phys. Rev. B **95**, 125130 (2017)

--- Relations with K matrix proposed for non-Abelian states? Ardonne, Bouwknegt, & Dawson (2003)

--- QH plateau transition or incompressible states from irrational/nontunitary CFT?

--- Realistic model for $SU(N)$ chiral spin liquids?