Model wavefunctions for Chiral Topological Order Interfaces

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Rencontres du Vietnam

Perspectives in Topological phases:
From Condensed Matter to High-Energy Physics

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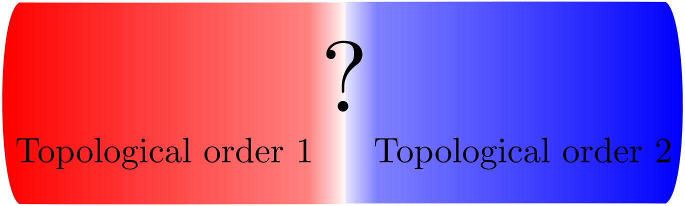
V. Crépel et al., arXiv :1806.06858
V. Crépel et al., PRB 97, 165136 (2018)

Motivations

What's going on at the interface between two topologically ordered phases?



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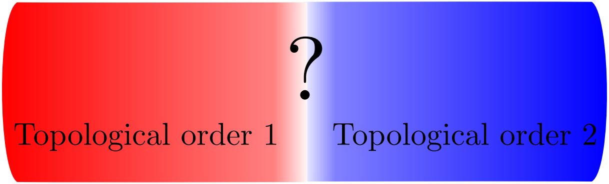

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Non-Abelian Anyons: When Ising Meets Fibonacci

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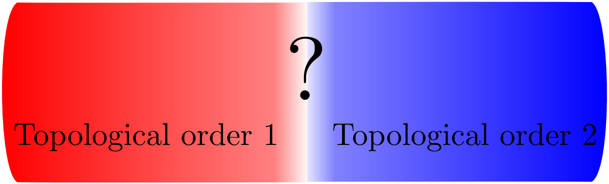
We consider an interface between two non-Abelian quantum Hall states: the Moore-Read state, supporting Ising anyons, and the $k = 2$ non-Abelian spin-singlet state, supporting Fibonacci anyons. It is shown that the interface supports neutral excitations described by a $(1 + 1)$-dimensional conformal field theory with a central charge $c = 7/10$. We discuss effects of the mismatch of the quantum statistical properties of the quasiholes between the two sides, as reflected by the interface theory.

Motivations

What's going on at the interface between two topologically ordered phases?



These predictions are based on effective low-energy approaches ("cut and glue"), in which one forgets about the bulk.

Two questions we want to address:

- Can we test these effective approaches ?
- Can we characterize the interface down to the microscopic level ?

$\Rightarrow$ **build accurate model wavefunctions for the full system (bulk+interface)**

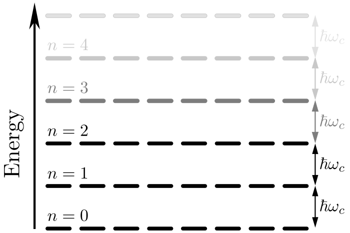
Outline

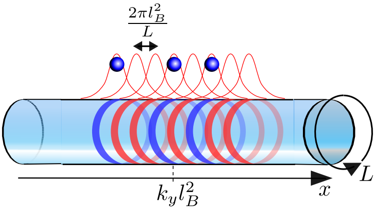
- FQH (Abelian) model states
- MPS for the FQH model states
- Building a variational Ansatz
- Characterizing the interface

FQH (Abelian) model states

Fractional Quantum Hall effect

Landau levels (spinless case)





- Cyclotron frequency : $\omega_c = \frac{eB}{m}$,
- Filling factor : $\nu = \frac{hn}{eB} = \frac{N}{N_\Phi}$
- Partial filling + interaction $\rightarrow$ FQHE
- Lowest Landau level $(\nu < 1)$: $z^m \exp\left(-|z|^2/(4l_B^2)\right)$
- N-body wave function : $\Psi = P(z_1, ..., z_N) \exp(-\sum |z_i|^2/(4l_B^2))$
- Landau gauge and cylinder : ring-like orbital centered around $k_y l_B^2$, k_y quantized.

The Laughlin wave function

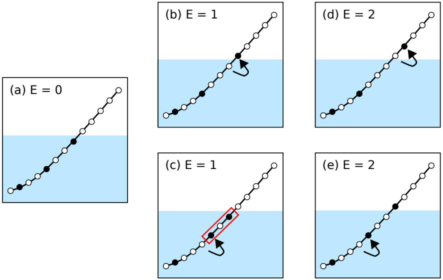
A (very) good approximation of the ground state at $\nu = \frac{1}{3}$

$$\Psi_L(z_1, ...z_N) \;=\; \prod_{i<j}(z_i - z_j)^3 e^{-\sum_i \frac{|z_i|^2}{4l^2}}$$

Excitations with fractional charge $\frac{e}{3}$ and fractional statistics

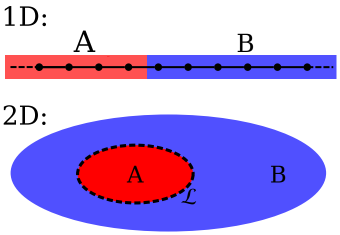
Edge excitations

- A chiral $U(1)$ boson with a dispersion relation $E \simeq \frac{2\pi v}{L} n$
- The degeneracy of each energy level is given by the sequence $1, 1, 2, 3, ...$



Entanglement entropy and entanglement spectrum

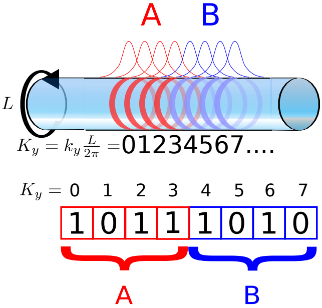
- Start from a quantum state $|\Psi\rangle$
- Create a bipartition of the system into A and B
- Reduced density matrix $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$
- Entanglement Hamiltonian : $\rho_A = e^{-H_{\mathrm{ent}}}$



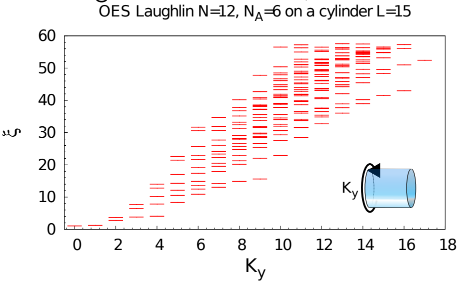
- The eigenvalues of H_{ent} are the entanglement energies $\{\xi_i\}$.
- Entanglement entropy $\mathcal{S}_A = -\mathrm{Tr}_A[\rho_A \ln \rho_A]$, area law for gapped systems.
- 2d topological phase $\rightarrow \quad \mathcal{S}_A \sim \alpha L - \gamma$.
- Topological term $\gamma = \ln\left(\frac{\mathcal{D}}{d_a}\right)$, a signature of the topological order.

FQH : (Orbital) entanglement spectrum

- FQHE on a cylinder (Landau gauge): orbitals are labeled by k_y, rings at position $\frac{2\pi k_y}{L} l_B^2$
- Divide your orbitals into two groups A and B, keeping $N_{\mathrm{orb},A}$ orbitals : orbital cut $\simeq$ real space cut (fuzzy cut)



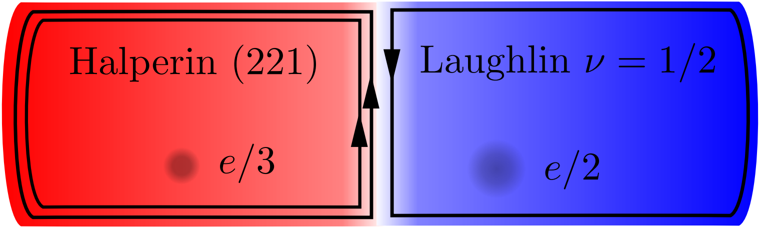
Laughlin state $N = 12$, half cut



- Fingerprint of the edge mode (edge mode counting) can be read from the ES. ES mimics the chiral edge mode spectrum.
- For FQH model states, nbr. levels is exp. lower than expected.

A simple (still rich) example

We consider the (bosonic) case interfacing the Halperin (221) and the Laughlin $\nu = 1/2$ phases.



Halperin (221)
$SU(3)_1$

- Spinful, $\nu_\uparrow = \nu_\downarrow = 1/3$.
- $e/3$ excitations.
- Two $U(1)$ chiral edge modes (charge and spin).

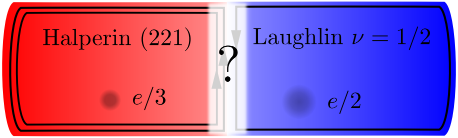
Laughlin $\nu = 1/2$
$SU(2)_1$

- Spinless, $\nu_\uparrow = 0, \nu_\downarrow = 1/2$.
- $e/2$ excitations.
- One $U(1)$ chiral edge mode (charge).

A microscopic model

$$\mathcal{H}_{\rm int} = \int \mathrm{d}^2\vec{r} \left(\sum_{\sigma,\sigma'=\uparrow,\downarrow} :\rho_\sigma(\vec{r})\rho_{\sigma'}(\vec{r}): \right) + \mu_\uparrow(\vec{r})\rho_\uparrow(\vec{r})$$

- Use the chemical potential $\mu_\uparrow(\vec{r})$ to polarize half of the system.
- Laughlin $\nu = 1/2$ is the interaction densest polarized zero energy state.
- Halperin (221) is the interaction densest unpolarized zero energy state.
- The two quantum liquids are sewed together by the interaction.



What shall we observe at the interface ? A single gapless mode described by a free chiral boson (Haldane, PRL 94).

MPS for the FQH model states

Model states and CFT

- A large set of model wavefunctions can be written as a CFT correlator (Laughlin, Moore-Read, Read-Rezayi, Halperin...).

$$\Psi(z_1, \cdots, z_N) \;=\; \langle V(z_1) \cdots V(z_N) \mathcal{O}_{\mathrm{bg}} \rangle$$

  with electron operator $V(z)$ in some chiral $1+1$ CFT and $\mathcal{O}_{\mathrm{bg}}$ is the background charge.

- Bulk-edge correspondence : The CFT used to describe the (gapped) bulk is identical to the CFT that describes the (gapless) edge
- **Laughlin state** : $SU(2)_1$
  - $V(z) =: \exp(i\sqrt{m}\Phi(z)) :$, where $\Phi(z)$ is a free chiral boson
  - $\langle \Phi(z_1)\Phi(z_2) \rangle = -\log(z_1 - z_2)$
  - $\langle V(z_1) \cdots V(z_N) \rangle = \prod_{i<j} (z_i - z_j)^m$
- **Halperin state** : Two free chiral bosons ($SU(3)_1$).

Matrix Product States

Any state can be written as

$$|\Psi\rangle = \sum_{\{m_i\}} \langle \alpha_L| A^{[m_1]} \ldots A^{[m_{N_{\rm orb}}]} |\alpha_R\rangle \; |m_1, \ldots, m_{N_{\rm orb}}\rangle$$

- $\{A^{[m]}\}$ is a set of $\chi \times \chi$ matrices
- (α_l, α_r) encode the boundary conditions for an open system.

The $A^{[m]}_{\alpha,\beta}$ matrices have two types of indices

- $[m]$ is the physical index ($m \in \{0, 1\}$ for fermions, $m \in \mathbb{N}$ for bosons, $m \in \{\uparrow, \downarrow\}$ for spins ...)
- (α, β) are the bond indices (auxiliary space), ranging from $1, \ldots, \chi$.
- The **bond dimension** χ is of the order of $\exp S_A$
  $\Rightarrow$ for 2d gapped phases, it grows exponentially with L.
  An exponential improvement over the $\exp(\mathrm{surface})$ of ED...

Starting from a model wavefunction given by a CFT correlator

$$\Psi(z_1, \cdots, z_N) = \langle u|\mathcal{O}_{\mathrm{b.c.}} V(z_1) \cdots V(z_N)|v\rangle$$

and expanding $V(z) = \sum_n V_{-n} z^n$, one finds (up to orbital normalization)

$$c_{(m_1,\cdots,m_n)} = \langle u|\, \mathcal{O}_{\mathrm{b.c.}} \frac{1}{\sqrt{m_n!}} V_{-n}^{m_n} \cdots \frac{1}{\sqrt{m_2!}} V_{-2}^{m_2} \frac{1}{\sqrt{m_1!}} V_{-1}^{m_1} \,|v\rangle$$

This is a site/orbital dependent MPS

$$c_{(m_1,\cdots,m_n)} = \langle u|\, \mathcal{O}_{\mathrm{b.c.}} B^{[m_n]}(n) \cdots B^{[m_2]}(2) B^{[m_1]}(1) \,|v\rangle$$

with matrices at site/orbital j (including orbital normalization)

$$\boxed{B^{[m]}(j) = \frac{e^{\left(\frac{2\pi}{L} j\right)^2}}{\sqrt{m!}} \left(V_{-j}\right)^m}$$

Translation invariant MPS

A relation of the form $B^{[m]}(j) = U^{-1} B^{[m]}(j-1) U$ yields

$$B^{[m]}(j) = U^{-j} B^{[m]}(0) U^{j}$$

and then

$$B^{[m_n]}(n) \cdots B^{[m_1]}(1) = U^{-n} \times B^{[m_n]}(0) U \cdots B^{[m_1]}(0) U$$

This is a **translation invariant MPS**, with matrices

$$A^{[m]} = B^{[m]}(0) U$$

Translation invariant MPS on the cylinder

Site independant MPS

$$B^{[m]}(j) = \frac{e^{\left(\frac{2\pi}{L} j\right)^2}}{\sqrt{m!}} \left(V_{-j}\right)^m \qquad \Rightarrow \qquad A^{[m]} = \frac{1}{\sqrt{m!}} \left(V_0\right)^m U$$

where U is the operator is (Zaletel and Mong (2012))

$$U = e^{-\frac{2\pi}{L} H - i\sqrt{\nu}\varphi_0}$$

where

- φ_0 is the bosonic zero mode ($e^{-i\sqrt{\nu}\varphi_0}$ shifts the electric charge by ν)
- H is the CFT cylinder Hamiltonian : $H = \frac{2\pi}{L} L_0$
- V_0 is the zero mode of $V(z)$

auxiliary space = CFT Hilbert space
infinite bond dimension :/

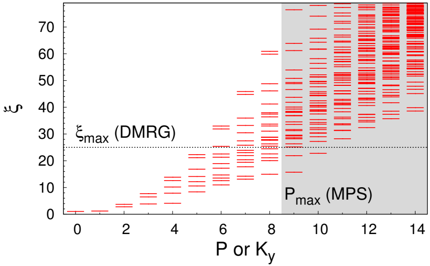
Extension to spinfull FQH : **V. Crépel et al., PRB 97, 165136 (2018)**

Truncation of the auxiliary CFT basis

- The natural cut-off is the total conformal dimension $\rightarrow P_{\mathrm{max}}$.
- Truncation over the momentum in the OES.
- In finite size, the truncated MPS becomes exact for P_{max} large enough.

- DMRG : cut-off in ξ (remove the smallest weight of ρ_A).
- MPS : cut-off in momentum.

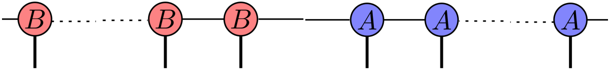
Equivalent if the ES mimics the chiral edge mode spectrum.



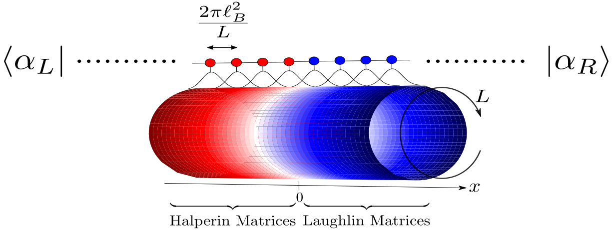
Building a model state for the Halperin/Laughlin interface

MPS and variational Ansatz

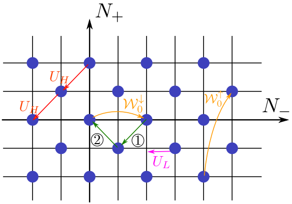
- We know the exact MPS for Halperin $B^{[n]}$ and Laughlin $A^{[n]}$.



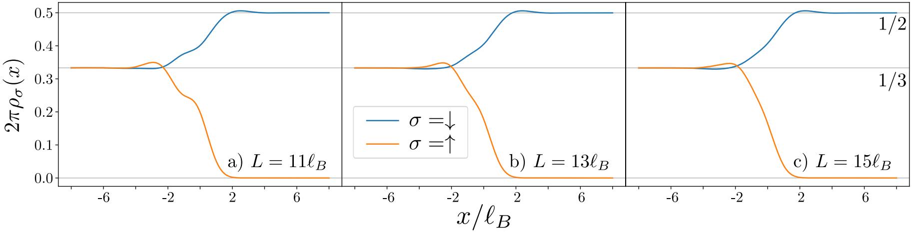
- Brutal gluing : $\langle \alpha_L | \cdots {\color{red}B^{[m_{-2}]}}{\color{red}B^{[m_{-1}]}}{\color{blue}A^{[m_0]}}{\color{blue}A^{[m_1]}} \cdots | \alpha_R \rangle$



- Does $B^{[m_{-1}]}A^{[m_0]}$ make any sense?
- Yes : **conformal embedding !**
- More generally $SU(2)_k \hookrightarrow SU(3)_k$

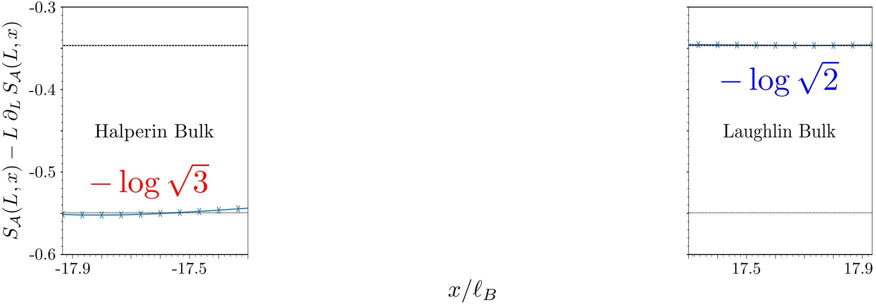


Density



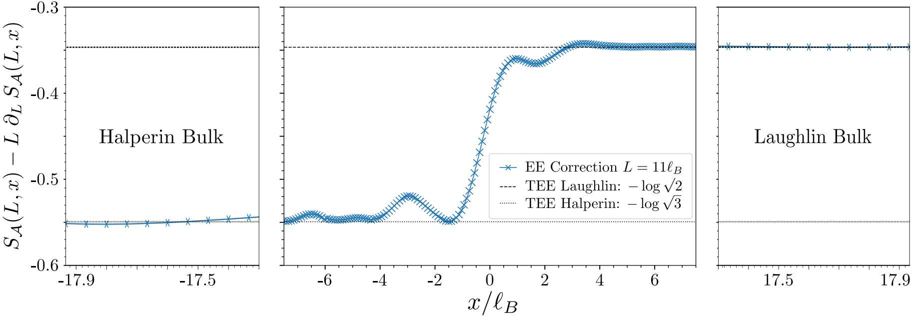
- Translation invariance along the cylinder perimeter.
- We recover the spin up and down densities in the bulk both on the Halperin and Laughlin side.
- Finite size effects (with respect to L) quickly vanish.
- Width of interface $\simeq 5 l_B$

Topological Entanglement Entropy



- We extract the TEE γ from the derivative $S_{\mathcal{A}} - L\partial_L S_{\mathcal{A}} = -\gamma$.
- Good agreement with the predicted values deep in the bulks (-0.549 and -0.347).

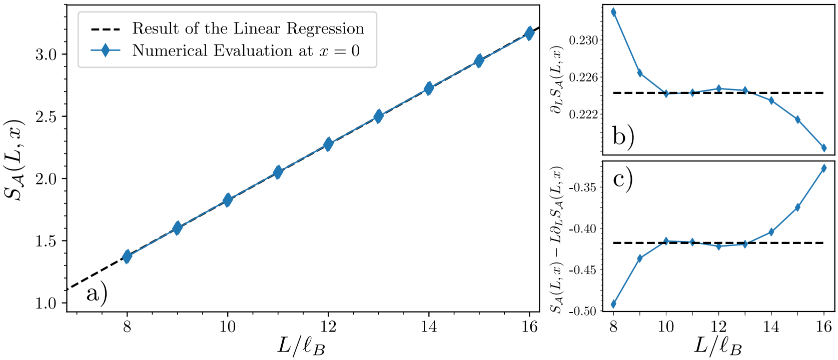
Topological Entanglement Entropy



- Up to a small oscillations (finite size effects more important for subleading terms), a rather smooth transition between the two bulk TEE.
- No sign of the gapless mode (as recently predicted by Santos et al. arXiv:1803.04418).

Area law at the transition

Does we still satisfy the area law at the interface?

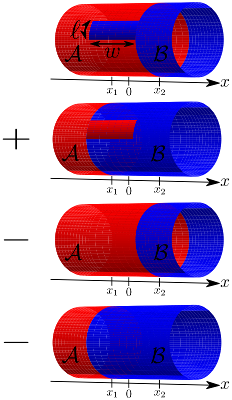


Yes (but hard to spot any deviation with such a limited range).

Characterizing the interface

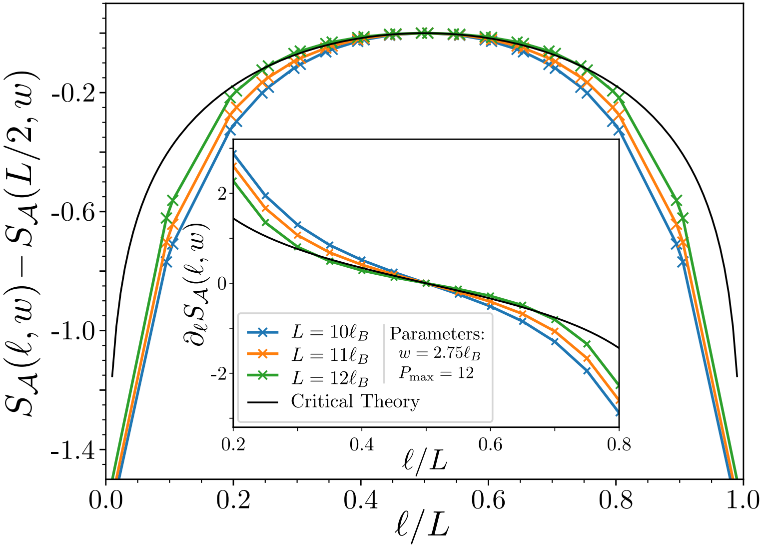
Extracting c : Levin-Wen cut

$$\underbrace{\alpha(x_1)\ell + \alpha(x_2)(L-\ell) + 2\int_{x_1}^{x_2} \alpha(u)\mathrm{d}u}_{\text{Area Law}} + \underbrace{\frac{c}{6}\log\left[\sin\left(\frac{\pi\ell}{L}\right)\right]}_{\text{Critical Mode}} + K(w)$$



- $K(w)$ contains corrections to the area law, corner contributions,...
- Using a Levin-Wen cut to focus on the critical contribution.
- $$S_{\mathcal{A}}(\ell, w) = 2\frac{c}{6}\log\left[\frac{L}{\pi}\sin\left(\frac{\pi\ell}{L}\right)\right] + f(w)$$
- To get rid of $f(w)$ (including the TEE), we compute $S_{\mathcal{A}}(\ell, w) - S_{\mathcal{A}}(L/2, w)$

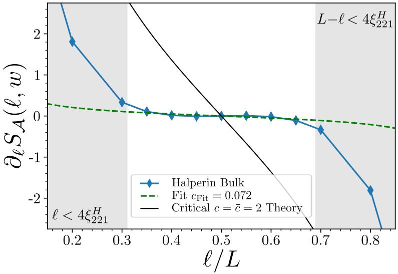
Extracting c : Levin-Wen cut



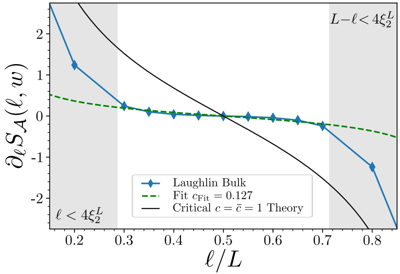
Fitted central charge $c = 0.987(1)$.

What about the bulk?

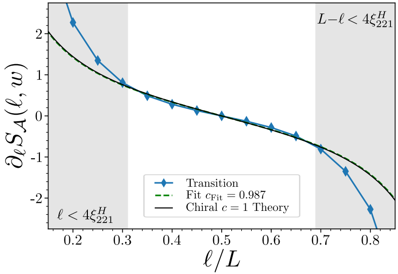
Halperin



Laughlin

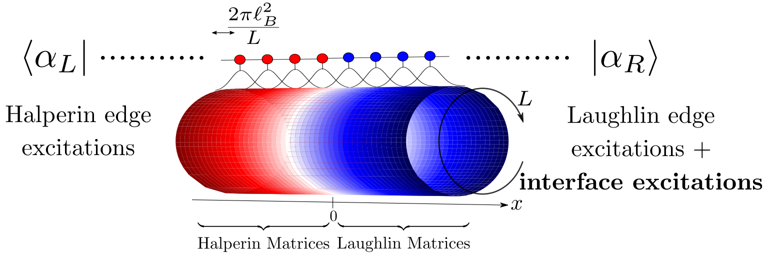


Interface

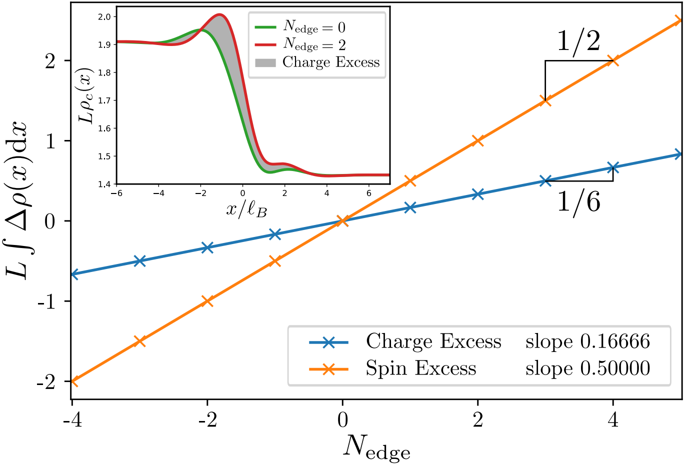


Compactification radius, fractional charge

- Central charge is only part of the information.
- Mutual information → full partition function of the CFT but hard to evaluate.
- Compactification radius ↔ edge mode charge.
- Directly measure the charge along the edge.
- Play with the MPS boundary conditions.



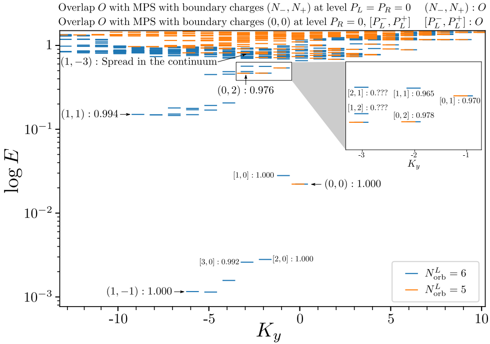
Compactification radius, fractional charge



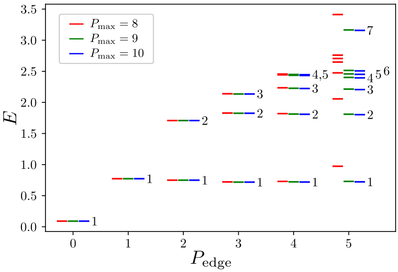
Excitations with a $e/6$ charge ($e/6 = e/2 - e/3$).

Is it a good variational wavefunction?

- It has all the features that we expect but *does it capture the microscopic model low energy properties?*
- Overlap with ED : $4\uparrow + 9\downarrow$ particles, 21 orbitals $\to$ 0.998 (Hilbert space dim $\simeq 2.2 \times 10^8$).



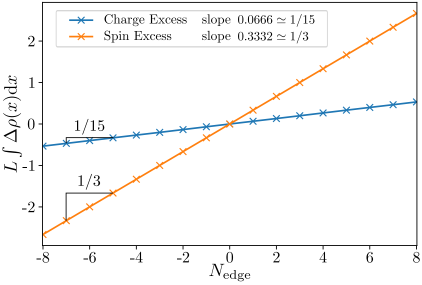
ED with 15 orbitals.

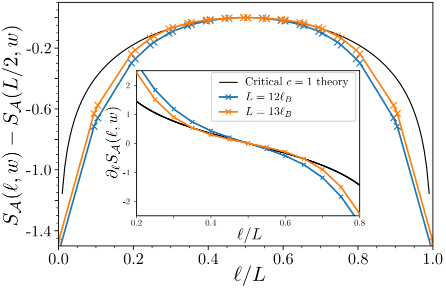


MPO $L = 12$.

Fermions : Laughlin $\nu = 1/3$ / Halperin (332)

- No conceptual difference with the bosonic example.
- Transition from Laughlin $\nu = 1/3$ to Halperin (332) at $\nu = 2/5$.
- Experimental relevance : graphene using the valley degree of freedom (spontaneous polarization at $\nu = 1/3$).





Conclusion

- A variational ansatz to describe the interface between the Halperin and the Laughlin liquids.
- Microscopic characterization of the interface gapless mode (c and R).
- This scheme can be extended to
  - any case where a MPS/PEPS/TN description is known on both sides. for instance

$$SU(2)_{k+l} \hookrightarrow SU(2)_k \otimes SU(2)_l$$

  - other sewing approach (e.g. superconductor).
  - a more generic approach ? couple the edge mode + numerical RG