Matrix Product State Description of Halperin States

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Quy Nhon, July 2018

PRB 97, 165136 (2018)
(see also arXiv:1806.06858)

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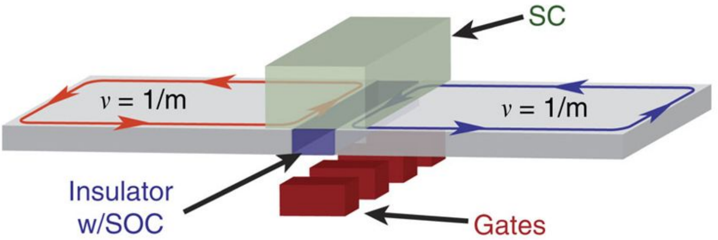
Motivations

- Many FQHE experimental realizations involve an internal degree of freedom

  Spin, Singlet-Triplet in QW, Graphene ... PRB 92, 075410 (Balram et *al.* - 2015)

- Bilayer systems: Non-Abelian excitation at the boundary between Abelian states

  Nat. Comm. 4, 1348 (Clarke et *al.* - 2013)



Nat. Comm. 4, 1348 (Clarke et *al.* - 2013)

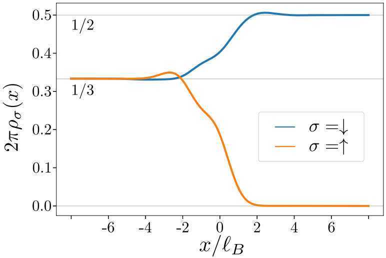
Motivations

- Quasi-electronic operators for FQH states shares some of the caveats

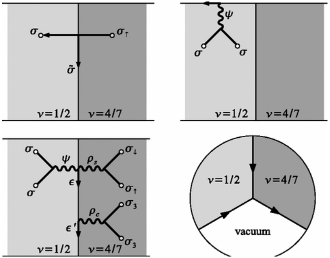
J. Stat. Mech. 053101 (Kjäll et *al.* - 2018)

- Use of conformal embedding $SU(2)_k \hookrightarrow SU(3)_k$ to create interfaces between topological orders (spinful problem - see B. Estienne's talk).

$k = 1$: Laughlin - Halperin
V.C. et *al.* arXiv:1806.06858



$k = 2$: MR - NASS
PRL 103, 076803 (Grosfeld and Schoutens 2009)



Halperin Wavefunctions

$$\psi_{mmn} = \prod_{i<j\leq N^{\downarrow}} (z_i^{\downarrow} - z_j^{\downarrow})^m \prod_{i<j\leq N^{\uparrow}} (z_i^{\uparrow} - z_j^{\uparrow})^m \prod_{i,j} (z_i^{\downarrow} - z_j^{\uparrow})^n$$

- Natural generalization of Laughlin WFs, recovered when $N^{\uparrow} = 0$
- Vanishing properties encoded in the $\mathbf{K}$-matrix

$$\mathbf{K} = \begin{pmatrix} m & n \\ n & m \end{pmatrix}$$

PRB 46, 2290 (Wen and Zee - 1992)

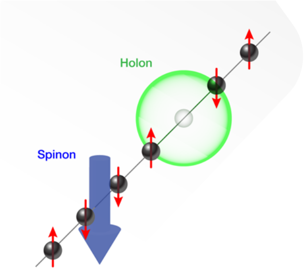
- ψ_{mmn} represents the spatial WF on component $|\uparrow \cdots \uparrow\downarrow \cdots \downarrow\rangle$. Full WF $|\Psi_{mmn}\rangle$ requires (anti-)symmetrization over both spin and position.
- Can be extended to any larger number of components.

MPS Derivation 1 - CFT

The Halperin states can be written as CFT correlators.
CFT matches the one describing the critical edge.

Nucl. Phys. B 360,2 (Moore and Read - 1991)

$$\psi_{mmn} = \langle 0|\mathcal{O}_{\text{bkg}}\mathcal{W}^{\uparrow}(z_1^{\uparrow})\cdots\mathcal{W}^{\uparrow}(z_{N\uparrow}^{\uparrow})\mathcal{W}^{\downarrow}(z_1^{\downarrow})\cdots\mathcal{W}^{\downarrow}(z_{N\downarrow}^{\downarrow})|0\rangle$$



- Two-component Luttinger liquid: spin-charge separation
- Vertex operators (factorization of $\mathbf{K}$ + OPE)

$$\mathcal{W}^{\uparrow}(z) =: e^{i\sqrt{\frac{m+n}{2}}\varphi^c(z)+i\sqrt{\frac{m-n}{2}}\varphi^s(z)}:$$

$$\mathcal{W}^{\downarrow}(z) =: e^{i\sqrt{\frac{m+n}{2}}\varphi^c(z)-i\sqrt{\frac{m-n}{2}}\varphi^s(z)}:$$

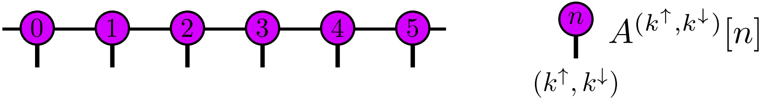
Remark: Additional $SU(2)_1$ algebra on φ_s for spin singlet states (*i.e.* $n = m - 1$).

MPS Derivation 2 - Modes and Symmetrization

Step 1: Mode expansion $\mathcal{W}^{\sigma}(z) = \sum_{n\in\mathbb{Z}} z^n W^{\sigma}_{-n} \Rightarrow W^{\sigma}_{-n}$ corresponds to an electron of spin σ on orbital n. Estienne et *al.* arXiv:1311.2936

Step 2: Correct statistics with explicit Klein factor at the CFT level $\chi = e^{i\pi\sqrt{2(m+n)}a_0^c} = (-1)^{R_c a_0^c}$. Numerically efficient!

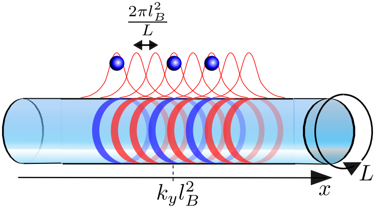
$$A^{(k^{\uparrow},k^{\downarrow})}[n] = \frac{1}{\sqrt{k^{\uparrow}!k^{\downarrow}!}} \left(W^{\uparrow}_{-n}\chi\right)^{k^{\uparrow}} \left(W^{\downarrow}_{-n}\right)^{k^{\downarrow}}$$



MPS Derivation 3 - iMPS

Cylinder geometry and iMPS

PRB 86, 245305 (Zaletel and Mong - 2012)



Spread background charge equally between orbitals and reproduce geometrical factors:

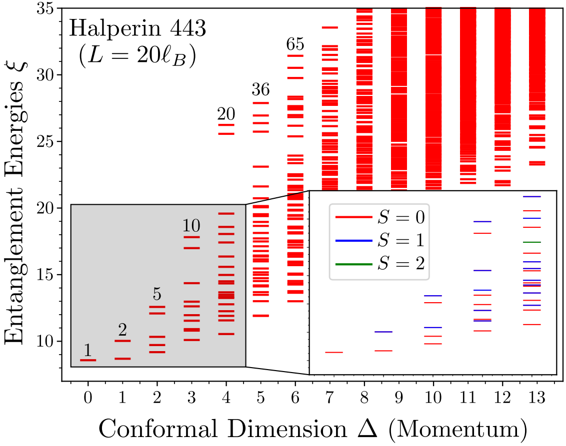
$$B^{(k^\uparrow, k^\downarrow)} = A^{(k^\uparrow, k^\downarrow)}[0]U$$

$$U = e^{-i\sqrt{\frac{2}{m+n}}\varphi_0^c} \; e^{-\left(\frac{2\pi}{L}\right)^2 L_0}$$

L_0 is the Hamiltonian on a circle, states becomes irrelevant with increasing energy. Natural truncation of infinite matrices *wrt* the conformal dimension (as in Truncated Conformal Space Approach of RG).

Numerical Implementation and Entanglement Entropy

Another motivation for this truncation: it preserves the exact CFT structure.

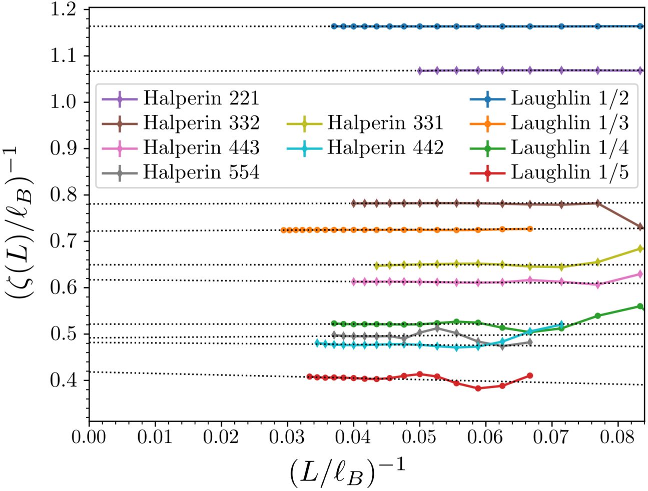


- Counting of two-component free boson 1-2-5-10-20-...
- Exact SU(2) symmetry (Ward identities) Even at finite truncation!
- Exact structure requires more states than usual DMRG procedure

Numerical Results: Correlation Lengths

Gapped? Yes, it may describe a Hall liquid.

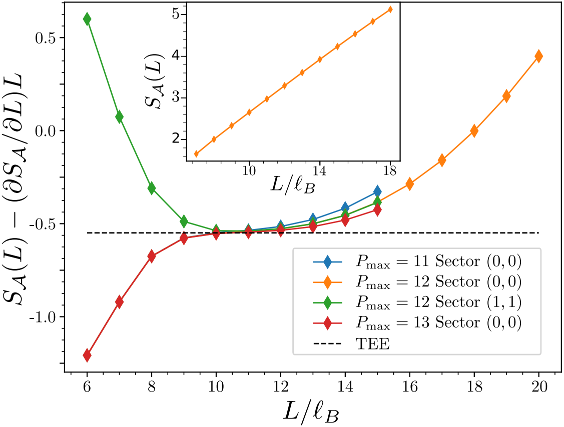
Decay of any generic correlator, necessary for braiding/adiabatic arguments!



Numerical Results: Topological Entanglement Entropy

A fully developed opological order? Extract the correction to the area law $S_{\mathcal{A}}(L) = \alpha L - \gamma$ which characterizes the topological order.

PRL 96, 110404 (2006) ; PRL 96, 110405 (2006)



- We extract the TEE γ from the derivative $S_{\mathcal{A}} - L\partial_L S_{\mathcal{A}} = -\gamma$
- Extracted $\gamma \simeq 0.545(5)$ agrees with theory $\log\sqrt{\det \mathbf{K}} = 0.549...$
- Abelian: Same γ for all topo sectors

Conclusion

- We derived an MPS to describe the Halperin states and all their zero energy excitations.
- Solved symmetrization issue (requirement for multicomponent continuous MPS!) and reached an infinite MPS form by spreading the background charge.
- Imposed a <u>global</u> SU(2) symmetry by a careful choice of truncation.
- Numerical characterization of the phase topological content, and evaluation of the correlation lengths.

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Perspectives

- Composite fermion WF and Quantum Hall hierarchies. Symmetrization relies on the same Klein factors, and quasi-electronic operators derived

  J. Stat. Mech. 053101 (Kjäll et *al.* -2018)

- Microscopic characterization of interfaces. Transition between equivalent $\mathbf{K}$-matrices?

  Cano et *al.*, PRB 89, 115116

- Emergence of Moore-Read from polarization of a $1/2+1/2$ bilayer. Transition between the two (inhomogeneous polarization): Majorana trapped at the interface?

  PRB 93, 085115 (Liu et *al.* - 2016)

- Topologically distinct boundary conditions: interface Laughlin $1/2$-$1/4$ (parity of $e/4$ quasiholes number at the transition)?

Thank you for your attention!

Backup 1: Correlation Lengths